Urban Traffic Network Control by Distributed Satisficing Agents

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ABSTRACT
This work aims to present a distributed sliding-horizon control technique, applied to the control of an urban traffic network, where the control agents follow a satisficing approach coordinating themselves to obtain a solution that is satisfactory for all agents. The coordination mechanism finds, in a distributed way, the analytic center of the region where all agents are satisfied. We show that the analytic center is also Pareto optimal. Our approach is compared to the centralized one where, instead of a coordination mechanism, the solution is based on a fixed and ad hoc adjustment of the relative importance of the agents.

Categories and Subject Descriptors
G.1.6 [**Mathematics of Computing**]: Numerical Analysis—*Optimization*; I.2.11 [**Computing Methodologies**]: Artificial Intelligence—*Distributed Artificial Intelligence*; J.7 [**Computer Applications**]: Computers in Other Systems

General Terms
Performance, Design, Theory

Keywords
satisficing control, satisficing theory, multiagent control system, urban traffic control

1. INTRODUCTION
Traffic congestion, delays, and emissions of pollutants are recurring issues in dealing with urban traffic control and management [11]. Efforts to mitigate these problems are so diverse as the improvement and expansion of the existing traffic infrastructure, the implementation of control policies with priority for public transport, and the deployment of real-time traffic control systems. Examples of such control systems are PRODYN [6], OPAC [7], RHODES [10], and feedback control strategies based on the linear-quadratic regulator [4] and sliding-horizon control [3]. The feedback control strategies are inherently robust as has been observed in field studies [5]. Despite the increased performance of such control systems, the signaling control of traffic lights is carried out mostly using fixed-time control, in part due to its simplicity and mainly because of its low implementation and maintenance cost.

The present work aims to simplify the design, installation, and reconfiguration of sliding-horizon control techniques by carrying out sensing and control in a distributed manner, and also to improve the control performance using a *satisficing* multiagent system. In the satisficing approach, the agents try to attain a performance at least greater than their minimum specified level of performance but also, and very important, they coordinate themselves to obtain a solution that is satisfactory to all agents. The satisficing approach also allows negotiation between the agents when their minimum level of performance can not be simultaneously achieved. Although essentially different in its methods, our approach has a philosophical trace to Satisficing Theory [12] and Satisficing Control [8].

The contribution of this paper to the problem of traffic light control is twofold: one is the definition of a minimum level of performance and a negotiation policy that permit the agents to coordinate themselves, and the other is a mechanism for coordination. The coordination mechanism is to find, in a distributed way, the analytic center of the region where all agents are satisfied. We show that the analytic center is also Pareto optimal. Our approach is compared to the centralized one where, instead of a coordination mechanism, the solution is based on a fixed and ad hoc adjustment of the relative importance of the agents.

2. TRAFFIC DYNAMIC MODEL
In general, urban traffic networks are formed by junctions connected by road links where traffic lights may be used to coordinate the conflicting traffic flows. Among other possibilities [4], the traffic lights set the percentage of green time allocated to each link.

For this work, we will consider the network in Figure 1 with

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eight junctions and one traffic light at each link approaching a junction. We can see that the network of Figure 1 can easily be represented by the directed graph of Figure 2, where the nodes are the junctions $m \in \mathcal{M}$ and the links $(i, j) \in \mathcal{E} \subset \mathcal{M} \times \mathcal{M}$ are the arcs connecting the nodes. The state variable $x_{i,j}$ represents the number of vehicles (queue) in the link from node j to the affected node i.

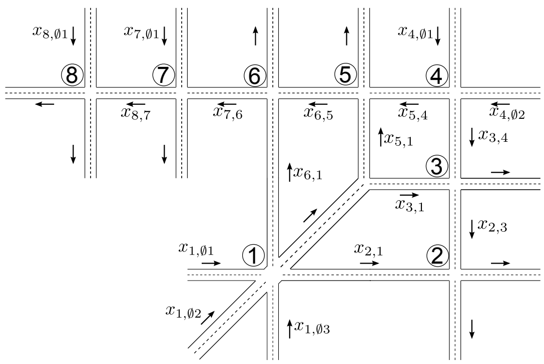

Figure 1: Example of a traffic network. The state variable $x_{i,j}$ is the number of vehicles (queue) of junction i affected by junction j.

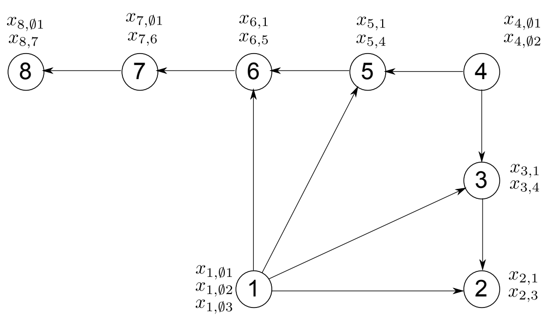

Figure 2: Graph for the traffic network example.

The interaction between nodes can be generalized assuming a generic node m and a set of input nodes $I(m) = \{i_1, ..., i_I\}$ and output nodes $O(m) = \{j_1, ..., j_O\}$ as in Figure 3. For example, node 3 has as input nodes $I(3) = \{1, 4\}$ and as output node $O(3) = \{2\}$. We divide the input nodes in internal and external nodes, for example $I(7) = I_{\mathrm{I}}(7) \cup I_{\mathrm{E}}(7)$ where $I_{\mathrm{I}}(7) = \{6\}$ is internal and $I_{\mathrm{E}}(7) = \{\emptyset 1\}$ is external.

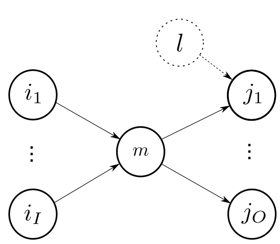

Figure 3: Illustration of input and output nodes of a node m.

The mathematical model chosen to describe the dynamics of the vehicle queues is based on the model known as store-and-forward [4] and is given by:

$$\mathbf{x}_m(k+1) = A_m \mathbf{x}_m(k) + \sum_{i \in I(m) \cup \{m\}} B_{m,i} \mathbf{u}_i(k) \qquad (1)$$

where the vector $\mathbf{x}_m(k) = (x_{m,i_1}(k), ..., x_{m,i_I}(k))$ are the queues of junction m influenced by the green time signals $\mathbf{u}_i(k) = (u_{i,i_1}(k), .., u_{i,i_I}(k))$ at instant k.

In this model, the matrix A_m is the identity, matrix $B_{m,m}$ expresses the discharge of queues $\mathbf{x}_m$ as a function of green times $\mathbf{u}_m$, and matrices $B_{m,i}$, $i \in I(m)$, represent how queues $\mathbf{x}_m$ build up as queues $\mathbf{x}_i$ are emptied by $\mathbf{u}_i$ green times. Matrices $B_{m,i}$, $i \in I(m) \cup \{m\}$, are functions of the physical characteristics of the traffic network. For the example, consider node 3 for which its matrices are:

$$B_{3,3} = T_3 \begin{bmatrix} -\frac{S_{3,1}}{C_3} & 0 \\ 0 & -\frac{S_{3,4}}{C_3} \end{bmatrix}$$

$$B_{3,1} = T_3 \begin{bmatrix} \rho_{3,1,\emptyset 1} \cdot \frac{S_{1,\emptyset 1}}{C_1} & \rho_{3,1,\emptyset 2} \cdot \frac{S_{1,\emptyset 2}}{C_1} & \rho_{3,1,\emptyset 3} \cdot \frac{S_{1,\emptyset 3}}{C_1} \\ 0 & 0 & 0 \end{bmatrix}$$

$$B_{3,4} = T_3 \begin{bmatrix} 0 & 0 \\ \rho_{3,4,\emptyset 1} \cdot \frac{S_{4,\emptyset 1}}{C_4} & \rho_{3,4,\emptyset 2} \cdot \frac{S_{4,\emptyset 2}}{C_4} \end{bmatrix}$$

where T_3 is the sample time (in seconds), $S_{i,j}$ is the saturation flow on the link from j to i (in vehicles per second), $\rho_{m,i,j}$ is the rate at which vehicles from link j to i enter link i to m, and C_i (in seconds) is the cycle time of junction i as explained below. Notice that the entries in $B_{3,3}$ are negative, indicating queue discharge as a function of green time signals $\mathbf{u}_3$.

The concept of cycle time is illustrated in Figure 4. Each cycle is composed by stages meaning a particular traffic light configuration. In the example of Figure 4, after stage 3, stage 1 repeats starting another cycle. From one stage to another there is a lost time added to avoid interference between stages. The sum of all green times plus lost times in a junction gives the cycle time for that junction.

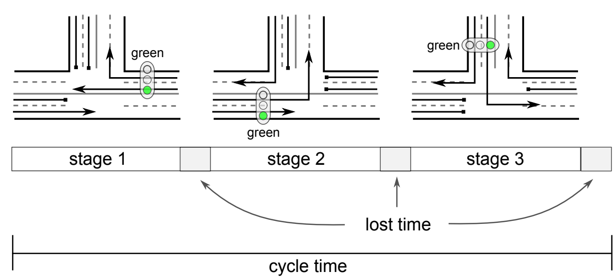

Figure 4: Illustration of the cycle time.

Three constraints are imposed to the junctions:

Constraint 1: The sum of the green times $u_{m,i}$ and lost time $L_{m,i}$ must be equal to the cycle time C_m of the junction m to which they belong,

$$\sum_{i \in I(m)} u_{m,i} + L_{m,i} = C_m, \quad \forall m \in \mathcal{M}$$

Constraint 2: The green times can not be negative,

$$\mathbf{u}_m \geq 0, \quad \forall m \in \mathcal{M}$$

Constraint 3: The states are always nonnegative,

$$\mathbf{x}_m \geq 0, \quad \forall m \in \mathcal{M}$$

3. DISTRIBUTED SATISFICING CONTROL

We propose a distributed approach to control the green time of the traffic lights, where a satisfactory global solution for the entire network is obtained from the specification of the agents. A convenient arrangement is to allocate one agent to each subsystem. In other words, for each node $m \in \mathcal{M} = \{1, \ldots, M\}$, the set of all nodes, we associate an agent $\mathcal{A}_m$ belonging to the agent set $\mathcal{A} = \{\mathcal{A}_1, \ldots, \mathcal{A}_M\}$.

Our agents apply a sliding-horizon control scheme where agent $\mathcal{A}_m$, $m \in \mathcal{M}$, calculates a plan of actions for N_m^{u} periods ahead of the current time so that the evolution in N_m^{p} periods of its states is satisfactory given a criterion. N_m^{u} and N_m^{p} are called control and prediction horizons respectively. Given the initial state $\mathbf{x}_m(0)$ of subsystem m, the predicted states are given by the following equation:

$$\begin{cases} \widetilde{\mathbf{x}}_m = \widetilde{A}_m \mathbf{x}_m(0) + \sum\limits_{i \in I(m) \cup \{m\}} \widetilde{B}_{m,i} \widetilde{\mathbf{u}}_i \\ \widetilde{\mathbf{y}}_m = \widetilde{C}_m \widetilde{\mathbf{x}}_m \\ \mathbf{y}_m(0) = C_m \mathbf{x}_m(0) \end{cases}$$

with

$$\widetilde{\mathbf{x}}_m = \begin{bmatrix} \mathbf{x}_m(1) \\ \mathbf{x}_m(2) \\ \vdots \\ \mathbf{x}_m(N_m^{\mathrm{p}}) \end{bmatrix}, \widetilde{\mathbf{y}}_m = \begin{bmatrix} \mathbf{y}_m(1) \\ \mathbf{y}_m(2) \\ \vdots \\ \mathbf{y}_m(N_m^{\mathrm{p}}) \end{bmatrix}, \widetilde{\mathbf{u}}_i = \begin{bmatrix} \mathbf{u}_i(0) \\ \mathbf{u}_i(1) \\ \vdots \\ \mathbf{u}_i(N_m^{\mathrm{u}} - 1) \end{bmatrix},$$

$$\widetilde{A}_m = \begin{bmatrix} A_m \\ (A_m)^2 \\ \vdots \\ (A_m)^{N_m^{\mathrm{p}}} \end{bmatrix}, \widetilde{C}_m = \begin{bmatrix} C_m & & \\ & \ddots & \\ & & C_m \end{bmatrix},$$

$$\widetilde{B}_{m,i} = \begin{bmatrix} I & \mathbf{0} & \cdots & \mathbf{0} \\ A_m & I & \cdots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ (A_m)^{N_m^{\mathrm{p}}-1} & (A_m)^{N_m^{\mathrm{p}}-2} & \cdots & (A_m)^{N_m^{\mathrm{p}}-N_m^{\mathrm{u}}} \end{bmatrix} B_{m,i}$$

where vector $\widetilde{\mathbf{x}}_m$ is the predicted states N_m^{p} periods ahead, and vector $\widetilde{\mathbf{u}}_m = (\mathbf{u}_m(0), \mathbf{u}_m(1), \ldots, \mathbf{u}_m(N_m^{\mathrm{u}} - 1))$ constitutes the *action plan* of agent $\mathcal{A}_m$, N_m^{u} periods ahead.

The prediction model can be restated in a compact form as:

$$\widetilde{\mathbf{x}}_m = \widetilde{A}_m \mathbf{x}_m(0) + \widetilde{B}_m \widetilde{\mathbf{v}}_m \tag{2}$$

where $\widetilde{B}_m = \begin{bmatrix} \widetilde{B}_{m,m} & \widetilde{B}_{m,i_1} & \cdots & \widetilde{B}_{m,i_I} \end{bmatrix}$ and the vector $\widetilde{\mathbf{v}}_m = (\widetilde{\mathbf{u}}_m, \widetilde{\mathbf{u}}_{i_1}, \cdots, \widetilde{\mathbf{u}}_{i_I}) = (\widetilde{\mathbf{u}}_m, \widetilde{\mathbf{u}}_{I(m)})$ is called the *plan profile* of agent $\mathcal{A}_m$, N_m^{u} periods ahead.

3.1 Agent Behavior

Our goal through the following subsections is to model an appropriate behavior for the agents so as to accomplish their objectives and restrictions and to guarantee, in a distributed way, individual and global specified criteria of performance. The satisficing theory [13, 12] proposes that the objectives of the agents should be evaluated using two indexes, one related to the goals (which therefore should be maximized) and the other related to the cost of energy or resources (which therefore should be minimized). These indexes will serve as a basis for establishing the *behavior* of the agents.

Definition 1. Selectability *(f_S) is the index of the usefulness of the actions with respect to the objective.*

Definition 2. Rejectability *(f_R) is the index of the cost associated with the actions.*

While the selectability function is normally concave since the usefulness of the actions should be maximized, the rejectability function is typically convex because the cost of the actions should be minimized.

Selectability and rejectability are the building blocks for the utilities of the types of agents that will be considered in the sequel, namely *selfish* and *satisficing* agents.

3.2 Selfish Agents

Defining the symmetric matrices $\widetilde{Q}_m \succeq 0$ (positive semidefinite) and $\widetilde{R}_m \succ 0$ (positive definite) as:

$$\widetilde{Q}_m = \begin{bmatrix} Q_m & & \\ & \ddots & \\ & & Q_m \end{bmatrix}, \ \widetilde{R}_m = \begin{bmatrix} R_m & & \\ & \ddots & \\ & & R_m \end{bmatrix}$$

of appropriated dimensions, we define the selectability and the rejectability of agent $\mathcal{A}_m$ by the following functions:

$$f_{S,m} = -\widetilde{\mathbf{x}}_m' \widetilde{Q}_m \widetilde{\mathbf{x}}_m \tag{3}$$

$$f_{R,m} = \widetilde{\mathbf{u}}_m' \widetilde{R}_m \widetilde{\mathbf{u}}_m \tag{4}$$

The selfish utility of agent $\mathcal{A}_m$ is given by:

$$f_m(\widetilde{\mathbf{v}}_m, \mathbf{x}_m(0)) = f_{S,m} - \alpha_m f_{R,m}, \ \alpha_m > 0$$

that depends on its plan profile and on its initial state (initial queues). From now on we will omit the dependence of the utility on the initial state.

The maximization of the selfish utility expresses the desire of the agent to maintain the queues and the green times close to zero, the stable equilibrium.

3.3 Satisficing Agents

In a cost versus benefit analysis, any action that results in an acceptable selectability compared to the rejectability is a defensible choice that belongs to the satisficing set.

Definition 3. Satisficing Set*: the region S of the domain where the difference between selectability and an adjusted rejectability results more than a minimum acceptable level of utility, formally*

$$S = \{\widetilde{\mathbf{v}} | f(\widetilde{\mathbf{v}}) = f_S(\widetilde{\mathbf{v}}) - \alpha \cdot f_R(\widetilde{\mathbf{v}}) \geq \beta\}$$

with $\alpha \in [0,\infty)$ denoting the sensitivity to cost with respect to the benefit and $\beta \in \mathbb{R}$ being the minimum acceptable level of utility, or satisfaction*. Any solution that belongs to the satisficing set is a* satisficing solution*.*

The objective of the satisficing agent $\mathcal{A}_m$ is to find a satisficing solution, where its selfish utility f_m results greater than a minimum level of satisfaction β_m. For the urban traffic application, we define

$$\beta_m = N_m^{\mathrm{p}}(-\mathbf{x}_m^{\mathrm{s}'} Q_m \mathbf{x}_m^{\mathrm{s}}) \tag{5}$$

where $\mathbf{x}_m^{\mathrm{s}}$ are the maximal, but still satisfactory, average queues of junction m. This definition of β_m expresses the desire of the agent to maintain the average queues less than the satisfactory maximal queue.

3.4 Coordination of the Satisficing Agents

In a multiagent system, the agents should coordinate themselves to reach a satisfactory collective solution.

The classical way to define a global utility H for a set of M agents is through a scalarization approach [1] according to which the interests of selfish agents are aggregated as a function

$$H(\widetilde{\mathbf{v}}) = \sum_{m=1}^{M} w_m f_m(\widetilde{\mathbf{v}}_m), \qquad w_m > 0, \forall m \in \mathcal{M}$$

of the vector $\widetilde{\mathbf{v}} = (\widetilde{\mathbf{u}}_1, \ldots, \widetilde{\mathbf{u}}_M)$. The decision process is centralized and defined by the following problem over $\widetilde{\mathbf{v}}$:

$$P^{\mathrm{C}} : \begin{cases} \text{Maximize} & H(\widetilde{\mathbf{v}}) = \sum_{m=1}^{M} w_m f_m(\widetilde{\mathbf{v}}_m) \\ \text{subject to} & \widetilde{\mathbf{v}} \in \widetilde{\mathcal{D}} \end{cases} \tag{6}$$

where $\widetilde{\mathcal{D}}$ is a generic convex domain. One characteristic of this problem is that any optimal solution $\widetilde{\mathbf{v}}^\star$ to problem P^{C} is Pareto optimal. Another characteristic is that the adjustment of w_m defines a particular solution in the Pareto set.

In a distributed system, on the other hand, a solution is reached by the interactions of the agents. Selfish agents produce a Nash point that is normally not Pareto. Instead, we will apply the satisficing agents which solve the following satisficing problem:

$$P_m^{\mathrm{S}} : \begin{cases} \text{find} & \widetilde{\mathbf{u}}_m \in \widetilde{\mathcal{D}}_m \\ \text{such that:} & f_m(\widetilde{\mathbf{v}}_m) \geq \beta_m \\ & f_j(\widetilde{\mathbf{v}}_j) \geq \beta_j, \forall j \in O(m) \end{cases} \tag{7}$$

where each agent $\mathcal{A}_m$ tries to find a satisficing solution for itself and for the affected agents.

The interactions of the satisficing agents will produce a solution in the jointly satisficing set,

$$S \triangleq \{\widetilde{\mathbf{v}} : f_m(\widetilde{\mathbf{v}}_m) \geq \beta_m, \forall m \in \mathcal{M}\}$$

Notice that in this case we have not just one solution but a set of possible solutions.

THEOREM 1. *The analytic center $\widetilde{\mathbf{v}}^\dagger = (\widetilde{\mathbf{u}}_1^\dagger, \cdots, \widetilde{\mathbf{u}}_M^\dagger)$ of the satisficing set $S = \{\widetilde{\mathbf{v}} : f_m(\widetilde{\mathbf{v}}_m) \geq \beta_m, \forall m \in \mathcal{M}\}$ is Pareto optimal and equivalent to the centralized solution with $w_m = 1/(f_m(\widetilde{\mathbf{v}}_m^\dagger) - \beta_m)$.*

We call coordination the process by which the agents, in a distributed manner, try to find a jointly satisficing solution and, in particular, the analytic center of the satisficing set.

The constrained analytic center of the satisficing set is obtained in two phases. In phase I a feasible solution $\widetilde{\mathbf{v}}^{\mathrm{s}}$ for which $f_m(\widetilde{\mathbf{v}}^{\mathrm{s}}) \geq \beta$ for all $m \in \mathcal{M}$ is calculated and used in phase II for the computation of the analytic center $\widetilde{\mathbf{v}}^\dagger$ of the satisficing set.

The phase I problem of agent $\mathcal{A}_m$ is of the form:

$$\begin{aligned} F_m^{\mathrm{I}}(\widetilde{\mathbf{u}}_{\neg m}) : \min_{\widehat{\mathbf{u}}_m} \ & F_m^{\mathrm{I}}(\widehat{\mathbf{u}}_m | \widetilde{\mathbf{u}}_{\neg m}) = \sum_{j \in \widetilde{O}(m)} s_j \\ \text{s.t.} : \ & f_m(\widetilde{\mathbf{u}}_m | \widetilde{\mathbf{u}}_{\neg m}) \geq \beta_m - s_m \\ & f_j(\widetilde{\mathbf{u}}_m | \widetilde{\mathbf{u}}_{\neg m}) \geq \beta_j - s_j, \ \forall j \in O(m) \\ & s_j \geq 0, \ \forall j \in \tilde{O}(m) \\ & \widetilde{\mathbf{u}}_m \in \widetilde{\mathcal{D}}_m \end{aligned} \tag{8}$$

where $\widehat{\mathbf{u}}_m = (\widetilde{\mathbf{u}}_m, \widetilde{\mathbf{s}}_m)$, $\widetilde{\mathbf{s}}_m = (s_j : \forall j \in \widetilde{O}(m))$, $\widetilde{O}(m) = O(m) \cup \{m\}$ and $\widetilde{\mathbf{u}}_{\neg m}$ denotes the action plan of the agents but agent $\mathcal{A}_m$. Let $\widehat{\mathbf{v}}^{\mathrm{I}} = (\widehat{\mathbf{u}}_1^{\mathrm{I}}, \ldots, \widehat{\mathbf{u}}_M^{\mathrm{I}})$ be an optimal solution to the set of problems $\{F_m^{\mathrm{I}}\}_{m \in \mathcal{M}}$. If $F_m^{\mathrm{I}}(\widehat{\mathbf{v}}^{\mathrm{I}}) = 0$ for all $m \in \mathcal{M}$, then the satisficing set is nonempty. If $F_m^{\mathrm{I}}(\widehat{\mathbf{v}}^{\mathrm{I}}) > 0$ for any m, then there does not exist a simultaneously satisficing solution for all the agents, in which case $\{m \in \mathcal{M} : s_m > 0\}$ is the subset of agents that cannot be satisfied.

Phase II solves, starting from the solution found in phase I, the set $\{F_m\}_{m \in \mathcal{M}}$ of problems given by:

$$\begin{aligned} F_m(\widetilde{\mathbf{u}}_{\neg m}) : \ & \min_{\widetilde{\mathbf{u}}_m} F_m(\widetilde{\mathbf{u}}_m | \widetilde{\mathbf{u}}_{\neg m}) = \\ & - \sum_{j \in \widetilde{O}(m)} \log(f_j(\widetilde{\mathbf{u}}_j | \widetilde{\mathbf{u}}_{\neg j}) - \beta_j) \\ & \text{subject to} \ \ \widetilde{\mathbf{u}}_m \in \widetilde{\mathcal{D}}_m \end{aligned} \tag{9}$$

where the functions $F_m(\widetilde{\mathbf{u}}_m | \widetilde{\mathbf{u}}_{\neg m})$, called log barrier functions [1], force the solution to the analytic center of the satisficing set.

Phase I and II problems can be solved by the agent set $\mathcal{A}$ using a distributed interior-point method. The convergence analysis presented in [2] ensures that the iterative solution of the set of problems in phase I and II converges to the analytic center when only nonneighboring agents iterate in parallel.

In every cycle, the coordination of the agents is achieved by the calculation of the analytic center of the problem set $\{P_m^{\mathrm{S}}\}_{m \in \mathcal{M}}$. According to Theorem 1, the analytic center results in a Pareto solution. This Pareto solution is equivalent to that obtained by a centralized solution *if w_m where not fixed*. However, the centralized approach uses fixed weights that are adjusted in an ad hoc manner.

3.5 Negotiation

From the definition of the satisficing set it can be seen that a smaller sensitivity α and/or a smaller satisfaction β lead to bigger satisficing sets. So, when the agents can not be simultaneously satisfied, they have to negotiate adjusting their sensibility to cost or adjusting their minimum level of satisfaction.

4. SIMULATION RESULTS

This section presents the application of distributed satisficing agents to the control of the 8-junction urban traffic network shown in Figure 1. The experimental analysis aims to assess the performance of the distributed satisficing control approach by comparing it to a centralized approach. The satisficing agents solve the set $\{P^{\mathrm{S}}_m\}_{m\in\mathcal{M}}$ of analytic center problems and a centralized agent solves P^{C}, while respecting the constraints described in Section 2.

The centralized problem and the analytic center of the satisficing problems were simulated in `Matlab` and solved using `CVX` [9], a package for specifying and solving convex programs.

4.1 Experimental Setup

There are only four parameters that must be defined by the user, with the advantage that all of them have physical meaning. They are:

P.1) The satisfactory maximal average queues $\mathbf{x}^{\mathrm{s}}_m$: an average queue that still is acceptable at each link of junction m.

P.2) The capacity of the links: the maximum number of vehicles that the link can support.

P.3) The prediction horizon N^{p}_m.

P.4) The cycle time C_m.

For this simulation, the satisfactory maximal average queue was defined as two times the maximal discharge obtained by the nominal green time set as $\mathbf{u}^{\mathrm{nom}}_1 = (40, 40, 40)$ and $\mathbf{u}^{\mathrm{nom}}_{2..8} = (60, 60)$, that is, $\mathbf{x}^{\mathrm{s}}_m = -2B_{m,m}\mathbf{u}^{\mathrm{nom}}_m$ for all $m \in \mathcal{M}$. The capacity of the links were set as three times the satisfactory queue, but in a real application the capacity is easily assessed based on the dimensions of the link. The prediction horizon was chosen equal to 5 minutes (300 seconds) and the cycle time equal to 2 minutes (120 seconds), for all agents.

The remaining parameters were set according to the following rules:

R.1) $T_m = C_m$: the sample time T_m was made equal to the cycle time for all agents.

R.2) $N^{\mathrm{u}}_m = N^{\mathrm{p}}_m$: the horizons were made equal in all junctions.

R.3) Matrix $R_m = 0$: in our simulation traffic signaling does not incur any cost. Green times should not be penalized.

R.4) Matrix $Q_m = \mathrm{diag}(1/\mathrm{cap}_{m,i} : i \in I(m))$: the matrix Q_m was set diagonal with its elements equal to the inverse of the capacity of each link that approaches junction m, as proposed in [4].

R.5) $\beta_m = N^{\mathrm{p}}_m(-\mathbf{x}^{\mathrm{s0}\prime}_m Q_m \mathbf{x}^{\mathrm{s0}}_m)$, where

$$x^{\mathrm{s0}}_{m,i} = \max(x^{\mathrm{s}}_{m,i}, x_{m,i}(0)) \quad \forall i \in I_{\mathrm{E}}(m)$$
$$x^{\mathrm{s0}}_{m,i} = x^{\mathrm{s}}_{m,i} \quad \forall i \in I_{\mathrm{I}}(m)$$

With the original definition of β_m (see Equation 5), an excessive number of vehicles coming from outside may become impossible for the system to maintain all the queues less that the satisfactory maximal average. In rule R.5, the original definition of β_m is modified to incorporate the negotiation policy chosen for the network. For this experiment, the negotiation policy is to maintain the specification of the internal nodes and degrade only the nodes receiving vehicles from outside the system allowing them to accumulate more vehicles. Observe that because nodes $2, 3, 5$ and 6 have only internal input nodes, their minimum level of satisfaction is not modified by rule R.5. On the other hand, nodes $1, 4, 7$ and 8 tolerate external queue sizes greater than the satisfactory, tolerating at least their actual number of vehicles, whatever it is.

The network physical parameters, saturation flows $S_{i,j}$ in vehicles per minute and conversion rates $\rho_{m,i,j}$ in percentage, are in the appendix.

4.2 Experimental Analysis

The analysis of the satisficing agents was made against a centralized one in which weights were set $w_m = 1$ for all $m \in \mathcal{M}$. Notice that the tuning of the centralized agent is based on an ad hoc definition of weights. We are considering a constant arrival of vehicles in node 1 and 4 equal to $(5, 15, 10)$ and $(10, 10)$ respectively, and initial queues $\mathbf{x}_1(0)$ through $\mathbf{x}_8(0)$ as $(10, 50, 20)$, $(60, 20)$, $(5, 35)$, $(120, 30)$, $(10, 20)$, $(9, 20)$, $(15, 20)$ and $(17, 9)$.

Figures 5 to 10 show the evolution in 10 cycles (20 minutes) of the vehicle queues, the calculated green times, and the utility of the satisficing and centralized agents, respectively. The bars show the vector components stacked from below. For example, in Figure 5, the components of the vector $\mathbf{x}_1 = (x_{1,\emptyset 1}, x_{1,\emptyset 2}, x_{1,\emptyset 3})$ are in black, gray and white respectively.

We can see in Figures 5 and 6 that the satisficing agents $\mathcal{A}_1$ and $\mathcal{A}_4$ accumulate more queues than the centralized agent although agent $\mathcal{A}_4$ starts to decrease its queues after a moment of increase. Something impedes $\mathcal{A}_1$ to maximize its time of green and also forces $\mathcal{A}_4$ to postpone in 1 cycle any relevant green time.

The reason is that agents $\mathcal{A}_1$ and $\mathcal{A}_4$ have a compromise with the satisfaction of the internal agents due to the solution of the satisficing problems and due to the negotiation scheme induced by rule R.5. Due to the negotiation rule, we can see in Figures 5 and 6 that agents $\mathcal{A}_1$ and $\mathcal{A}_4$ adjust their minimum level of satisfaction (dash-dot line) to permit lower utilities (solid lines) and to accommodate more queues. Remember that the policy we chose was to maintain the specification of the internal nodes and degrade only

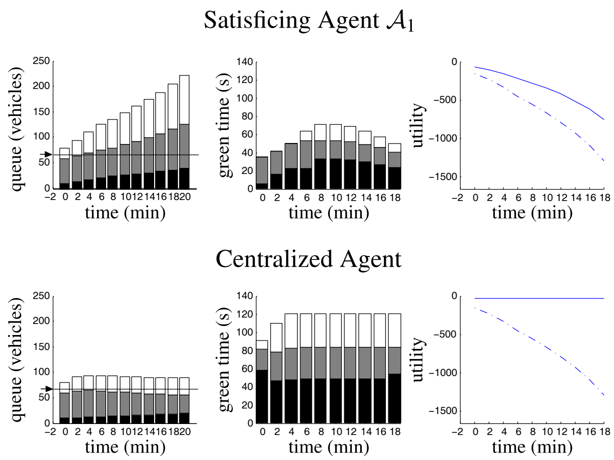

Figure 5: Control in junction 1.

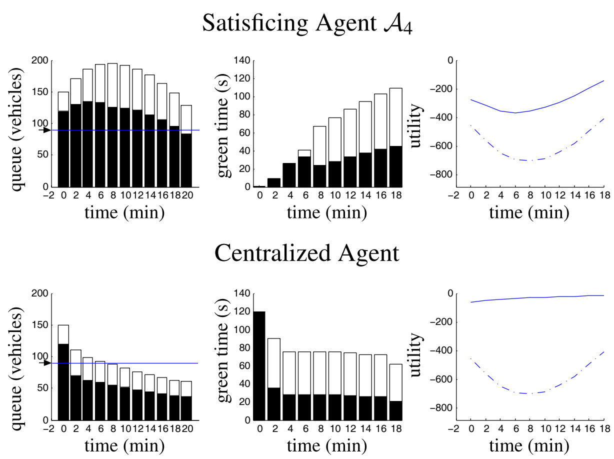

Figure 6: Control in junction 4.

the nodes receiving vehicles from outside the system. This compromise is difficult to obtain in the centralized control due to its ad hoc nature.

In Figures 7 and 8 we see that the discharge made by the centralized agent overcharges node 5 and node 6 making it unable to maintain the sum of the corresponding queues below the maximal satisfactory (horizontal line indicated by an arrow) even with the maximal of green (120 seconds). In the centralized case, the utility of nodes 5 and 6 are very below the minimum level specified for agents $\mathcal{A}_5$ and $\mathcal{A}_6$ (horizontal dash-dot line) indicating their dissatisfaction. We chose to show only the internal junctions 5 and 6 because they suffer the highest effect.

The satisficing agents also seem to present a better behavior in the presence of model error. Figures 9 and 10 show the behavior of junction 5 and 6 when the rates of flow coming from junction 4 are greater than what is expected by the nominal model. In this case, instead of 30%, the flow from junction 4 to junction 5 is 70% of the junction total flow. We see that the satisficing agents maintain the queues in a satisfactory level while the centralized agent builds up the queues even more.

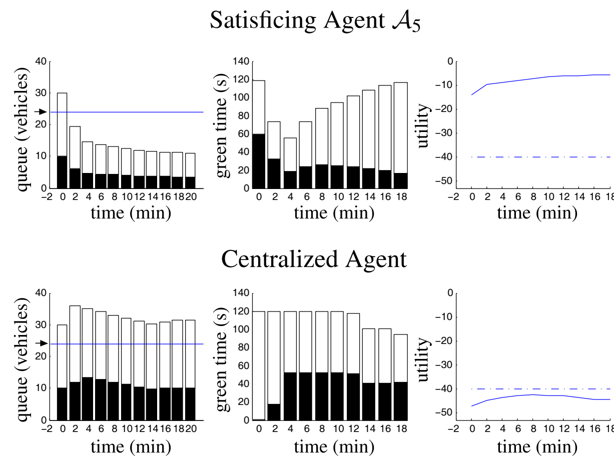

Figure 7: Control in junction 5.

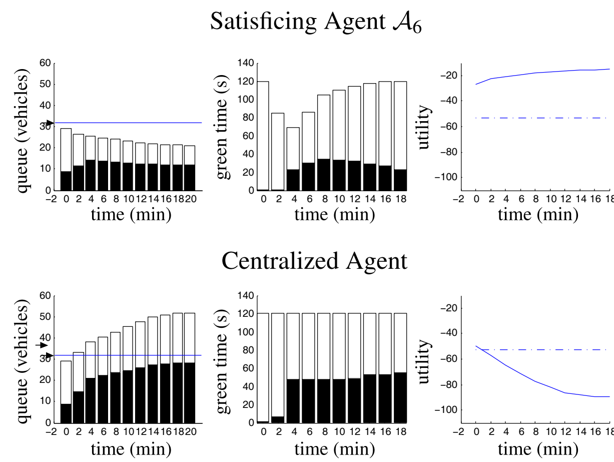

Figure 8: Control in junction 6.

5. CONCLUDING REMARKS

The satisficing approach offers a mechanism of coordination that, applied to an urban traffic network, has the following advantages if compared to a centralized classical approach:

- the adjustment of the agents are based on physical parameters instead of the ad hoc adjustment of weights;
- the definition of the minimum level of satisfaction gives meaning to the control objectives;
- the negotiation policy offers a mechanism to alleviate the control objectives in case of infeasibility and is flexible enough to accommodate other strategies. For example, instead of penalizing only the junction that receives vehicles from outside the system, one can define a negotiation policy where all the agents reduce their minimum level of satisfaction;
- and it seems to be more robust to model error.

It is also worth to mention that any satisficing solution, not only the analytic center, is good enough to make the agents satisfied. This fact can be used to simplify the distributed algorithm and reduce decision time.

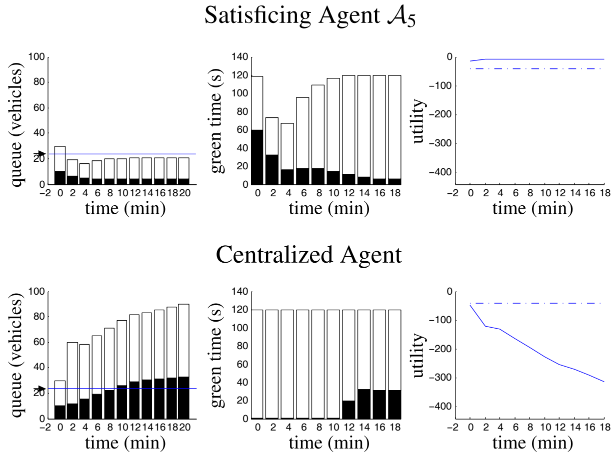

Figure 9: Junction 5 under model error.

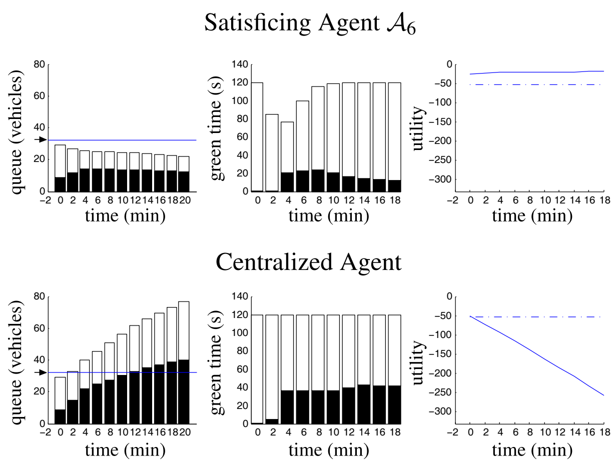

Figure 10: Junction 6 under model error.

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APPENDIX

A. PROOF OF THEOREM

THEOREM 1. *The analytic center* $\widetilde{\mathbf{v}}^\dagger = (\widetilde{\mathbf{u}}_1^\dagger, \cdots, \widetilde{\mathbf{u}}_M^\dagger)$ *of the satisficing set* $S = \{\widetilde{\mathbf{v}} : f_m(\widetilde{\mathbf{v}}_m) \geq \beta_m, \forall m \in \mathcal{M}\}$ *is Pareto optimal and equivalent to the centralized solution with* $w_m = 1/(f_m(\widetilde{\mathbf{v}}_m^\dagger) - \beta_m)$.

PROOF. (1) An optimal solution $\widetilde{\mathbf{v}}^\star = (\widetilde{\mathbf{u}}_1^\star, \cdots, \widetilde{\mathbf{u}}_M^\star)$ to the unconstrained centralized problem P^{C} : $\max_{\widetilde{\mathbf{v}}} H(\widetilde{\mathbf{v}})$, where $H(\widetilde{\mathbf{v}}) \triangleq \sum_{m=1}^{M} w_m f_m(\widetilde{\mathbf{v}}_m)$ and $w_m > 0$, is Pareto optimal. An optimal solution is obtained when $\nabla_{\widetilde{\mathbf{v}}} H(\widetilde{\mathbf{v}}^\star) = \mathbf{0}$, that is, when $\sum_{m=1}^{M} w_m \nabla f_m(\mathbf{v}_m^\star) = \mathbf{0}$, because H is concave. (2) On the other hand, the analytic center of the satisficing set S is obtained by solving the problem $\min_{\widetilde{\mathbf{v}}}\{F(\widetilde{\mathbf{v}}) = \sum_{m=1}^{M} -\log(f_m(\widetilde{\mathbf{v}}_m) - \beta_m)\}$. A solution is given by $\nabla_{\widetilde{\mathbf{v}}} F(\widetilde{\mathbf{v}}^\dagger) = \sum_{m=1}^{M} \frac{1}{f_m(\widetilde{\mathbf{u}}_m^\dagger) - \beta_m} \nabla f_m(\widetilde{\mathbf{u}}_m^\dagger) = \mathbf{0}$ because F is convex. From (1) and (2), it follows that the analytic center coincides with the solution obtained by solving the centralized problem with $w_m = 1/(f_m(\widetilde{\mathbf{v}}_m^\dagger) - \beta_m)$. $\square$

B. NETWORK PHYSICAL PARAMETERS

The network physical parameters, saturation S in vehicles per minute and direction rate ρ in percentage, are: $S_{1,\emptyset 1} = 5$, $S_{1,\emptyset 2} = 30$, $S_{1,\emptyset 3} = 15$, $S_{2,1} = 20$, $S_{2,3} = 25$, $S_{3,1} = 5$, $S_{3,4} = 25$, $S_{4,\emptyset 1} = 30$, $S_{4,\emptyset 2} = 15$, $S_{5,1} = 5$, $S_{5,4} = 7$, $S_{6,1} = 9$, $S_{6,5} = 7$, $S_{7,6} = 7$, $S_{7,\emptyset 1} = 10$, $S_{8,7} = 7$, $S_{8,\emptyset 1} = 5$, $\rho_{2,1,\emptyset 1} = 25$, $\rho_{2,1,\emptyset 2} = 70$, $\rho_{2,1,\emptyset 3} = 25$, $\rho_{2,3,1} = 40$, $\rho_{2,3,4} = 80$, $\rho_{3,1,\emptyset 1} = 7$, $\rho_{3,1,\emptyset 2} = 7$, $\rho_{3,1,\emptyset 3} = 20$, $\rho_{3,4,\emptyset 1} = 70$, $\rho_{3,4,\emptyset 2} = 70$, $\rho_{5,1,\emptyset 1} = 8$, $\rho_{5,1,\emptyset 2} = 8$, $\rho_{5,1,\emptyset 3} = 20$, $\rho_{5,4,\emptyset 1} = 30$, $\rho_{5,4,\emptyset 2} = 30$, $\rho_{6,1,\emptyset 1} = 60$, $\rho_{6,1,\emptyset 2} = 15$, $\rho_{6,1,\emptyset 3} = 35$, $\rho_{6,5,1} = 60$, $\rho_{6,5,4} = 90$, $\rho_{7,6,1} = 90$, $\rho_{7,6,5} = 90$, $\rho_{8,7,6} = 90$, $\rho_{8,7,\emptyset 1} = 50$.