Pedestrians' Route Choice Model for Shopping Behavior

Bruno Rocha Werberich Carlos Oliva Pretto Helena Beatriz Bettella Cybis
Department of Production Engineer
Federal University of Rio Grande do Sul
Porto Alegre, Brazil
{bruno.rwe;cpretto}@gmail.com , helenabc@producao.ufrgs.br

Abstract

This paper presents an agent-based model to address the pedestrian route choice problem in shopping malls. Route choice in shopping malls may be defined by a number of causal factors. Shoppers may follow a pre-defined schedule, they may be influenced by other people walking, or may want to get a glimpse of a familiar shopping. The route choice process assumes that the cost of each route can be calculated as a function of three factors: route length, impedance generated by other pedestrians and attraction for areas of interest on the environment. The impedance generated by the friction between pedestrians is assumed to exist even before physical contact, due to the psychological tendency to avoid passing close to individuals with high relative velocity. Pedestrians seek minimal route length and minimal friction with other pedestrians. In order to represent shopping areas environments, a new factor is being considered in the calculation of the route cost: the attraction for areas of interest on the environment. Simulation results were compared to real data collected by video recording in a shopping mall.

1 Introduction

Modelling of pedestrian's behavior is a complex task and has been studied by different research areas. In order to represent motion of pedestrians more realistically, models are required to simulate several processes, including sense and avoidance of obstacles, interaction with other pedestrians and route choice. Agent-based abstraction has been widely used for pedestrian modeling, mainly due to its capacity to provide insights about system´s reactions from changes on entities proprieties, capturing information over space and time at a detailed level [Klügl and Bazzan 2012; Macal et al. 2006; Rossetti R. et. al. 2002]. Agent-based models represent agents' decision-making ability based on their profile and perception over the environment.

Agent-based pedestrians models require the aggregation of different levels of abstraction, that are modeled on different layers. The majority of pedestrian models present a multi-layer simulation approach [Gaud et al. 2008; Hoogendoorn et al. 2002] composed by, at least, two layers: a tactical and an operational layer.

The tactical layer chooses a path regarding an origin-destination pair and a route choice criteria such as minimum distance and/or travel times. The tactical model determines the desired pedestrian directions, which are used in the operational model [Pretto et al. 2011].

The operational model determines the low level microscopic movements of pedestrians. It is ruled by principles of pedestrians'sense and avoidance of obstacles. Most models reported in literature can be regarded as using force-based approaches [Helbing et al. 1991; Helbing et al. 1995]. In force-based models, agents evaluate forces exerted by infrastructure and by other agents. Helbing and Molnar (1995) presented a relevant work on force-based models in which they use Newtonian mechanics and a continuous space representation to model a long-range interaction. The concept behind this approach suggests that the motion of a pedestrian can be described by combination of several forces (including the repulsive forces from walls and other pedestrians). The social force model reproduces various emergent phenomena observed on pedestrian´dynamics.

The tactical model is responsible for route choice. Realistic route choice is a complex process because most route selection strategies are based on subconscious decisions. Most models presented in the literature are concerned only with the quickest or shortest route, like Kirik et. al. (2009), Dressler et. al. (2010) and Lämmel et. al. (2014). However, other factors play an important role in route choice behavior, such as: peoples' habits, number of crossings, pollution and noise levels, safety, shelter from poor weather conditions and other environment stimulations [Papadimitriou E., 2012]. Most relevant route choice models are concerned with pedestrians' evacuation. In Kretz et. al. (2011), for instance, pedestrians routes are chosen based on the minimal remaining travel time to destination. Kretz et. al. (2014) introduce a generic method for dynamic assignment used with microsimulation of pedestrian dynamics. In the paper, the routes mark the most relevant routing alternatives in any given walking geometry, reducing the infinitely many trajectories by which a pedestrian can move from origin to destination to a small set of routes. Crociani and Lämmel (2016) present a work with two major topics. In the first topic, a novel cellular automaton (CA) model is proposed, which describes the pedestrian movement by a set of simple rules, and the second topic describes how the CA can be integrated into an iterative learning cycle where the individual pedestrian can adapt travel plans based on experiences from previous iterations. Patil et. al. (2010) propose an interactive algorithm to direct and control crowd simulation. The model presented by Treuille et. al., (2006) unifies route planning and local collision avoidance by using a set of dynamic potential and velocity. Teknomo (2008) and Teknomo et al., (2008) described a self-organization route choice approach to model the dynamics of agents, such as pedestrians and cars on a simple network graph. The agents decide, when reaching a vertex, which edge to enter next. This decision is based on a set of rules regarding the agent's observation of the local environment. In order to represent complex networks, such as shopping areas and urban scenarios, agents need to represent more complex caracteristics and capabilities.

The literature presents several agent-based applications to simulate different pedestrians' behaviors and environments. The pedestrians' simulation in a commercial environment, such as shopping malls, is particularly complex since pedestrians are exposed to different stimulus and attractions [Wang, W. et. al. 2014]. Agent-based simulation is particularly valuable for these cases because environment stimulus exert distinct influences depending on the person profile. Dijkstra et al., (2013) provide a model for pedestrian activity simulations in shopping environments. This framework provides an activity agenda for pedestrian agents, guiding their shopping behavior in terms of destination and time spent in shopping areas. Pedestrian agents need to successively visit a set of stores and move over the network. The authors assumed that pedestrian agents' behavior is driven by a series of decision heuristics. Agents need to decide which stores to choose, in what order and which route to take, subject to time and environment constraints.

Route choice in shopping malls may be defined by a number of causal factors. Shoppers may follow a pre-defined schedule, they may be influenced by other people walking, or may want to get a glimpse of a familiar shopping.

Shopping agents, as described in the literature [Borgers, A., and Timmermans, H., 1986; Ali, W. and Moulin, B., 2006] usually decide (i) in which stores to stop, (ii) in what order and (iii) which route to take. In practice, however, shopping mall users´ behaviour is a combination of planned and unplanned decisions. Planned decisions can defined by a set of origin-destination pairs. Unplanned decisions may be resultant from eventual impulses or the attraction exerted by shopping windows.

This paper presents an agent-based route choice model to represents pedestrians' in a shopping mall environment. The pedestrian model allows the representation of shopping users capable to perform either planned and unplanned behaviour, depending on the agent´s profile. Simulation results were compared to real data collected by video recording in a shopping mall.

2 The Model

An agent-based model is proposed to address pedestrian route choice problem. Agent-based models represent agents' decision-making ability based on agents' characteristics profile and perception over the environment. In the proposed model, pedestrians are agents able to choose and recalculate routes. Pedestrians are not assigned to predetermined routes.

In this model, a route is a set of coordinates followed by a pedestrian from origin to destination. Route choice process comprises three factors for calculation: (i) distance, (ii) interaction with other pedestrians (avoiding jams) and (iii) attraction for areas of interest on the environment (in this specific case: shop windows).

The framework adopted to describe pedestrian behavior in this model (Figure 1) presents a three-layer structure, each layer representing:

(i) Demand for travel - set of origin and destination. Each origin-destination pair is associated to a number of trips and a pedestrian generation rate. Origins and destinations are associated with nodes on the environment layer.

(ii) Simulation environment structure -.The environment is described as a continuous space and is composed by geometric entities, such as rooms, doors, and other obstacles. The environment entities are linked by a graph-based structure providing a route to all entities. In this model, nodes are defined by a set of coordinates (x, y). Nodes also contain properties defining local features of the environment.

(iii) Pedestrians movement, sense and avoidance of obstacles: set of equations and agents behavior rules. The social force model (1) describes pedestrian walking behavior regarding agents' low-level motion, collision avoidance and velocity adaptation. Pedestrians freely walk on the modeling environment seeking the next graph node of the designated route. Pedestrians' movements are ruled by the sense and avoidance model and are not restricted to a strict set of links.

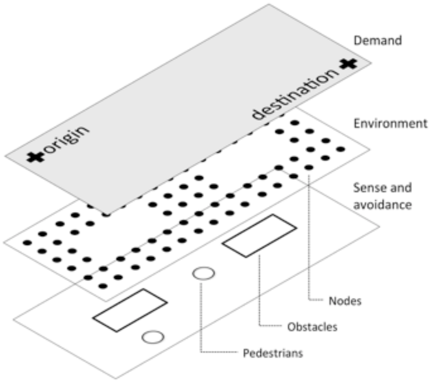

Figure 1 - Layers

2.1 The Route Choice Process

The presented route choice process is derivate from model established by Werberich et. al. (2014). Werberich et. al. propose that the cost of each route can be calculated as a function of two factors: route length and the impedance generated by other pedestrians. The impedance generated by the friction between pedestrians is assumed to exist even before physical contact, due to the psychological tendency to avoid passing close to individuals with high relative velocity [Helbing D. et al., 2000]. Pedestrians seek minimal route length and minimal friction with other pedestrians. In this model, a new factor is being considered in route cost calculation: attraction for areas of interest on the environment.

The total route cost is the sum of all link costs. Dijkstra algorithm [Dijkstra E., 1959] is adopted to generate valid routes for any origin/destination pair. Figure 2 describes the cost calculation for a link.

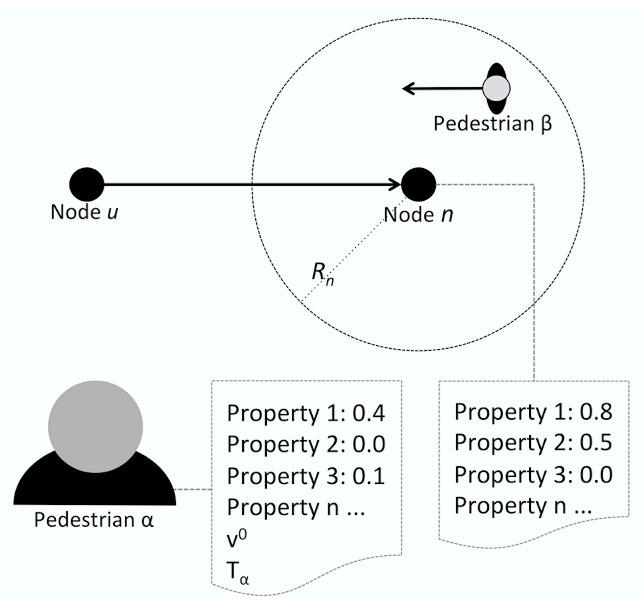

Figure 2 – Pedestrian's profile and node attraction

Figure 2 presents the elements involved in the route choice process. The cost estimation for a Pedestrian α to walk from node u to n involves three factors: (i) the distance between nodes ($\|\overrightarrow{r_n} - \overrightarrow{r_u}\|$), (ii) the impedance perceived by the pedestrian α exerted by other pedestrians (I_α) and (iii) the environment attraction perceived by pedestrian α for the node n (A_α^n).

Impedance exerted by the pedestrians in the simulation is calculated by simple vectors operations. Subtracting the desired velocity of pedestrian α from the velocity of pedestrians closer to node n (pedestrians β) it is possible to estimate I_α (equation 1).

$$I_\alpha = \sum_\beta \left| \overrightarrow{v_\beta} - \left(\frac{\overrightarrow{r_n} - \overrightarrow{r_u}}{\|\overrightarrow{r_n} - \overrightarrow{r_u}\|} \right) * v_\alpha^0 \right| \qquad (1)$$

where:

$\overrightarrow{v_\beta}$ = Pedestrian's β current velocity;
$\overrightarrow{r_n}$ = Node's n vector position;
$\overrightarrow{r_u}$ = Node's u vector position ;
v_α^0= Pedestrian's α desired speed.

The calculation of I_α considers a neighborhood area around the node n, defined by the radius R_n. All pedestrians inside the neighborhood area, at the instant of the route choice, are nominated pedestrians β. I_α is the sum of the friction forces exerted by each pedestrian β over the desired velocity of the pedestrian α.

As mentioned above, the graph nodes contain properties that classify local features of the environment. Node properties define the environment characteristics. For example, properties can be defined as female clothes store, male clothes store, electronics store, shoe store, etc. Nodes are defined by a set of values for all simulated properties. Higher properties values mean the node is closer of the related feature. Properties can assume values in the range [0 – 1].

The attraction exerted by these nodes properties on pedestrians vary dependeing on pedestrians profiles. Pedestrians' profiles also present a set of values for all simulated environment properties, that represent their attraction for these features. For example, male pedestrians probably have higher values for a property relating to a male clothes store. These properties also assume values in the range [0 – 1].

The attraction of node n, perceived by pedestrian α (A_α^n), is calculated as a weighted average (Equation 2):

$$A_\alpha^n = \frac{\sum_{i=0}^{p} P_i^\alpha * N_i^n}{\sum_{i=0}^{p} N_i^n} \qquad (2)$$

where:

p = total number of properties;
P_i^{α} = pedestrian α property i value;
N_i^n = node n property i value.

The total estimated cost for pedestrian α to walk from node u to n ($W_{\alpha}^{u,n}$), is a balance between distance, impedance and attractiveness, as described in Equation 3:

$$W_{\alpha}^{u,n} = \|\vec{r}_n - \vec{r}_u\| . (1 + I_{\alpha} / I_{max} + (1 - A_{\alpha}^{n})) \quad (3)$$

where:
I_{max} = settable parameter that adjusts the balance between distance and impedance. Further description of this parameter can be obtained in Werberich et al. (2014).

Elected routes minimize the total cost W_{α}. Equation 3 ensures pedestrians are attracted to areas of interest considering their profile. Pedestrians also avoid congested areas and passing close to other pedestrians with high relative velocity.

2.2 Pedestrian Stopping Behavior

It is expected that pedestrians walking on shopping environment, when attracted by an environmental stimulus, may stop for a while. For example, pedestrians attracted by a shop window frequently stop walking when they get closer to this interest point. This model simulates pedestrians route choice process subjected to attraction by interest areas, tipical of shopping environments.

To simulate pedestrians' stopping behavior the model introduces the concept of hotspots. Hotspots are defined by a location on the environment (x and y coordinates) and a neighborhood area (radius R). Hotspots have the same environment properties as graph nodes. When a pedestrian reaches the neighborhood area of a hotspot, he decides whether to stop or not. This decision process considers the pedestrian profile and the hotspot properties. Pedestrian profile includes a value denoting the tendency to stop on a hotspot (T_{α}). Higher values of T_{α} means the pedestrian have higher tendency to stop on hotspots. T_{α} values also respect the range [0–1]. Equation 4 defines the probability of a pedestrian α stopping on a hotspot q (S_{α}^{q}).

$$S_{\alpha}^{q} = \frac{\sum_{i=0}^{p} (P_i^{\alpha} * H_i^{q})}{\sum_{i=0}^{p} H_i^{q}} * T_{\alpha} \quad (4)$$

where:
p = total number of properties;
P_i^{α} = pedestrian α property i value;
H_i^n = hotspot q property i value;
T_{α} = pedestrian α tendency to stop on a hotspot.

If a pedestrian decides to stop on a hotspot neighborhood, the hotspot coordinates become his new destination for the stopping period. The balance between the pedestrian desired speed vector (v_{α}^{0}) and the forces exerted by the hotspot walls, keep the pedestrian standing in the neighborhood area. During this period, the interaction between pedestrians is maintained, allowing a realistic representation of pedestrians behavior at window shops. When a pedestrian stopping time has expired, a new route is recalculated to the final the destination.

The time a pedestrian stops at a hotspot may has variable assumptions. In this formulation, pedestrians stopping time is assumed to be fixed, equal to 20 seconds. Assumptions about stopping times can be discussed in more detail. An important work regarding time spent at store windows was developed by Dijkstra J. et. al. (2014). In this paper, authors describe the time spent in a store based on pedestrians profile and store segment.

Figure 3 presents a flowchart of the agent's internal process.

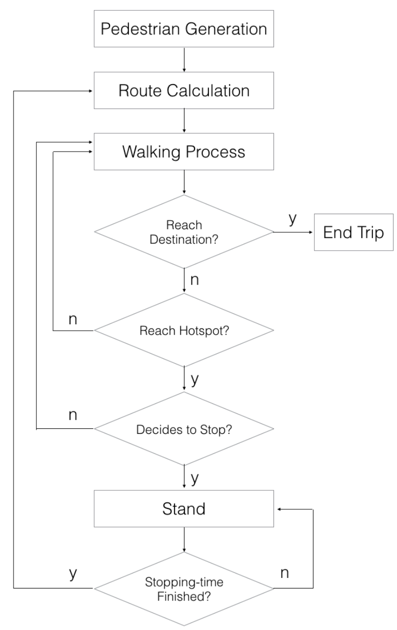

Figure 3 - Agent's internal process

As presented in this flowchart, a pedestrian only performs a route recalculation procedure after stopping at a hotspot. A Social Force-based route choice process considers the interaction with other pedestrians, which provides a dynamic behavior. However, if necessary, when simulating complex scenarios, the model structure allows the introduction of route recalculation areas. When simulating small scenarios, where the decision at the beginning of the trip was based on a good assessment of the way forward for all simulation timeframe, route recalculation may not be necessary.

3 Collected Data

Video data were collected in a shopping mall of Porto Alegre, Brazil. The camera collected images from a hall that connects the two main corridors of the first floor. Figure 4 presents an image of the studied area and the collected pedestrian routes.

The software *Tracker* was used to collect pedestrians' data in a semi-automatic process. The collected data is composed by a set of coordenates (*x and y*) over 1 minute of video for each pedestrian.

In order to simplify the data analysis, the enviroment was segmented in cells. A color map representing the cumulative occupation of each cell is shown at figure 5, segmented by gender.

Figure 4 – The Mall

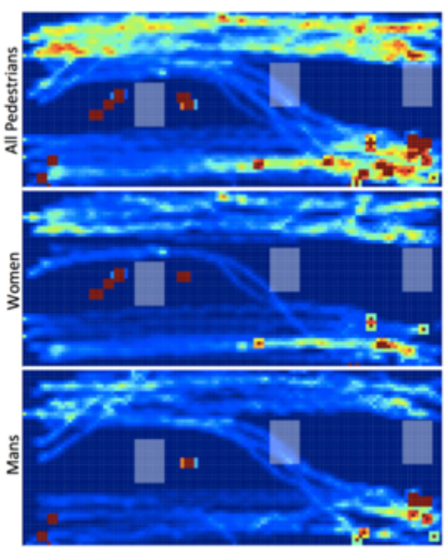

Figure 5 –Collected data

Data analysis allows the identification of three stores with higher pedestrian attraction . Table 1 shows the number of pedestrians, men (M) and women (W), that were attracted and stopped closer to these areas.

Table 1 – Stopped pedestrians

Store	M	W
jewelry	1	5
toy store	3	2
shoes store	2	3

4 Simulation

The proposed model has the potential to represent several properties regarding agents' profile and environment characteristics. In order to simplify the simulation, only two properties were considered in this experiment: Male Store Attraction (MSA) and Female Store Attraction (FSA). These two properties were applied to:

i. Scenario elements: hotspots and graph nodes (MSA_s and FSA_s);
ii. Agents (MSA_a and FSA_a).

The experiment was developed to identify the influence of MSAa and FSA_a in the number of pedestrians that are attracted to hotspots. The MSA_a and FSA_a were calibrated based on collected data.

The model was implemented using c# programming language (simulation engine) and Windows Presentation Foundation for the graphical interface.

4.1 Simulation Scenario

Figure 6 shows the simulation scenario built to represent the observed environment. Green areas (h1, h2, h3) are the hotspots. The hotspots correspond to stores where mall users used to stop on the real site. Dots are the graph nodes. Rectangles represent mall kiosks.

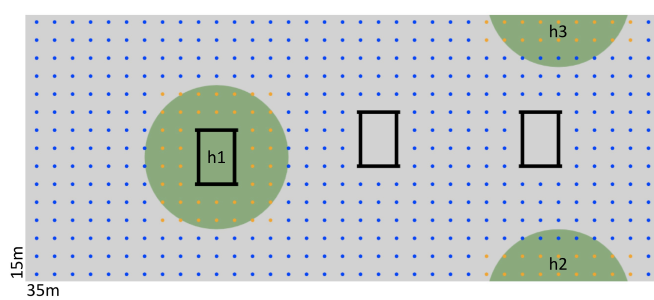

Figure 6 – Simulation scenario

Table 2 shows the values for MSA_s and FSA_s considered for the hotspots and its surrounding yellow graph nodes. Blue graph nodes (Figure 6) exert no attraction over the agent, the value for both MSA_s and FSA_s are zero. The MSA_s and FSA_s values were assumed to be constants. The MSA_s and FSA_s definition can be enhanced by considering effects of various design and management attributes. An example of the evaluation of consumers attraction can be found in Oppewal, H., and Timmermans, H. (1999). The authors estimated a stated preference model from responses to descriptions of an hypothetical shopping centers considering attributes such as: area for pedestrians, window displays, street layout, and street activities.

Table 2 – Hotspots configuration

hotspot	role	MSA_s	FSA_s
h1	jewelry	0.2	0.5
h2	toy store	0.8	0.6
h3	shoes store	0.8	0.6

4.2 Calibration

The calibration process aimed to calibrate the agents' profile (MSA_a and FSA_a) in order to reproduce the number of stopped pedestrians at each hotspot. For this purpose, four groups of simulations were run (s1, s2, s3, s4). For each simulation group, 50 simulations were performed. Two agents classes were implemented: male agents (MA) and female agents (FA). By definition, male agents have $FSA_a = 0$ and female agents have $MSA_a = 0$. Table 3 shows the configuration profiles defined for each simulation group.

Table 3 – Agents profile configuration

simulation group	MA	FA
s1	$MSA_a = 0.1$	$FSA_a = 0.1$
s2	$MSA_a = 0.5$	$FSA_a = 0.5$
s3	$MSA_a = 0.7$	$FSA_a = 0.7$
s4	$MSA_a = 0.9$	$FSA_a = 0.9$

The only variables in simulations were MSA_a and FSA_a. The scenario configuration was kept constant. Agents' tendency to stop (T_α) was set to 0.7. According to observed data, each simulation run comprised 80 agents, 40% MA and 60% FA. Pedestrians are generated with a fixed rate over time, with 40% of change to be male and 60% of change to be female. Figure 7 shows a simulation screenshot, MA are green circles and FA are red circles. A simulation video is availiable at:
https://youtu.be/10OUgNMaoNA.

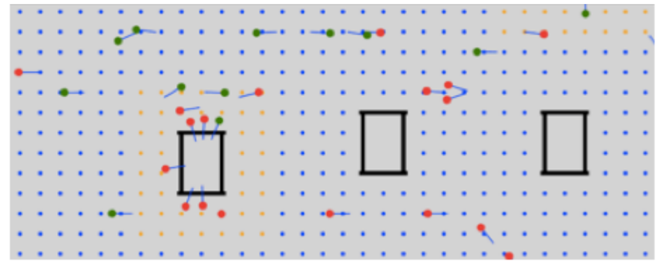

Figure 7 – Simulation screenshot

Figure 8 shows a color map of the results for all simulation groups (s1, s2, s3, s4), and the average number of agents stopping at each hotspot (h1, h2, h3) over 50 simulation runs.

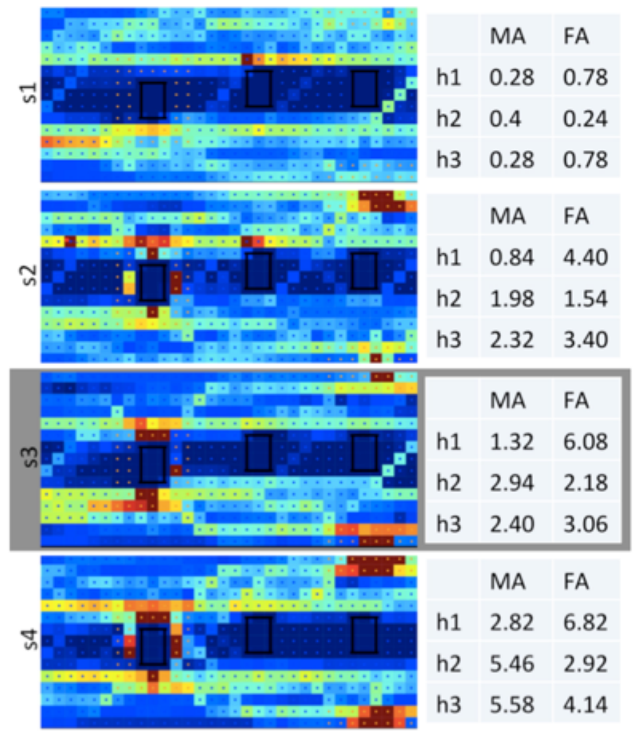

Figure 8 – Simulations results

4.2 Simularion Analysis

Simulation group s3 presented the best ajustment to the observed data. Higher values of MSA and FSA lead to higher attraction to hotspots. However, it is important to highlight that even though a pedestrian chooses a route to get closer to a shop window, he needs to reach a hotspot to stop. If the hotspot area is too crowded, he may not reach the hotspot, due to the social force effect, and do not stop. Thus, the attraction effect has a tendency to be balanced. Figure 9 show the s3 color map and the color map generated from real data. The s3 color map is one of 50 simulations. It is possible to observe differences in color patterns between simulation and real data. This difference is due the noise of pedestrians' tracking process and camera perspective. It is important to highlight stopping pattern at hotspots is similar.

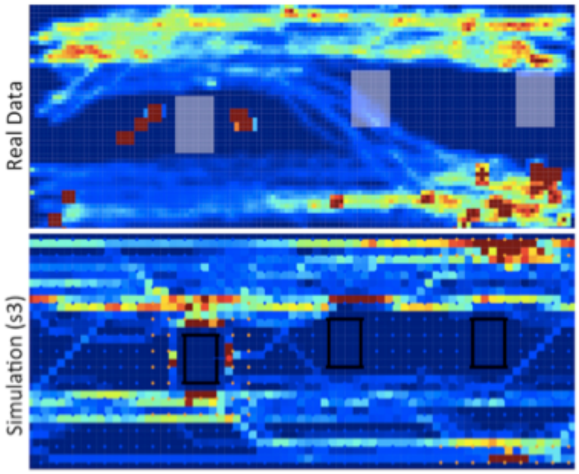

Figure 9 – Real data versus simulation data

5 Conclusions

The modeling approach presented in this paper provides a sound representation of pedestrian route choice dynamics considering the attraction to shop windows. Route choice is based on a combination of distance, impedance generated by other pedestrians and shop window attraction. The model differs from other pedestrians' route choice approaches because it seamlessly incorporates pedestrians social force into the route choice decision process.

In this model, we have created an association between the pedestrian's profile and store segment. When a pedestrian defines a route, due to its attraction to a store, he draws his chance to stop at a hotspot. The formulation of stopping chances can be enhanced through a more complex agent abstraction. However, it is well known that increasing model complexity usually leads to an increase in the calibration process effort.

The analysis from simulations indicates that the agents' emerging behavior provides a promising approach for real case applications. This model formulation is capable of supporting more complex agents' profiles and aplications to different enviroments, such as variable shopping premisses, expositions sites and passengers terminals.

Acknowledgments

The authors wants to thank the financial supported by the National Council for the Improvement of Higher Education (CAPES) and National Council for Scientific and Technological Development (CNPq), both of Brazil.

References

[Klügl, F., & Bazzan, A. L., 2012] Agent-based modeling and simulation. *AI Magazine*, *33*(3), 29.

[Macal, C. M., & North, M. J., 2006] Introduction to agent-based modeling and simulation. In *MCS LANS Informal Seminar*.

[Rossetti, R. J., Bordini, R. H., Bazzan, A. L., Bampi, S., Liu, R., & Van Vliet, D., 2002] Using BDI agents to improve driver modelling in a commuter scenario. *Transportation Research Part C: Emerging Technologies*, *10*(5), 373-398.

[Gaud N, Galland S, Gechter F, et al., 2008] Holonic multilevel simulation of complex systems: application to real-time pedestrians simulation in virtual urban environment. Simul Model Pract Theory 2008; 16: 1659–1676.

[Hoogendoorn S, Bovy P and Daamen W., 2002] Microscopic pedestrian wayfinding and dynamics modelling. In: Schreckenberg M and Sharma S (eds) Pedestrian and evacuation dynamics. Berlin: Springer, 2002, pp.123–155.

[Pretto CO, Jacobsen AC and Cybis H B B. 2011] A multi-layer simulation model for vehicle and pedestrian. In: Proceedings of the 90th Annual Meeting of the Transportation Research Board, Vol. 1, Washington, 2011, pp.1–15.

[Helbing D., 1991] A mathematical model for the behavior of pedestrians. Behav Sci. 1991; 36: 298–310.

[Helbing D and Molnar P., 1995] Social force model for pedestrian dynamics. Phys Rev E Stat Phys Plasmas Fluids Relat Interdisciplin Top 1995; 51: 42–82.

[Kirik ES, Yurgel'yan TB and Krouglov DV., 2009] The shortest time and/or the shortest path strategies in a pedestrian dynamics model. J Siberian Fed Univ Math Phys 2009; 2: 271–278.

[Dressler D, Groß M, Kappmeier J, et al. 2010] On the use of network flow techniques for assigning evacuees to exits. Procedia Eng 2010; 3: 205–215.

[Lämmel G and Plaue M., 2014] Getting out of the way: collision avoiding pedestrian models compared to the real world. In: Weidmann U, Kirsch U and Schreckenberg M (eds) Pedestrian and evacuation dynamics 2012. Berlin: Springer, 2014.

[Papadimitriou E., 2012] Theory and models of pedestrian crossing behaviour along urban trips. Transp Res Part F Traffic Psychol Behav 2012; 15: 75–94.

[Kretz T, Große A, Hengst S, et al., 2011] Quickest paths in simulations of pedestrians. Adv Complex Syst 2011; 14: 733–759.

[Kretz, T., Lehmann, K., Hofsäß, I., & Leonhardt, A., 2014] Dynamic assignment in microsimulations of pedestrians. arXiv preprint arXiv:1401.1308

[Crociani, L., & Lämmel, G., 2016] Multidestination Pedestrian Flows in Equilibrium: A Cellular Automaton-Based Approach. Computer Aided Civil and Infrastructure Engineering, 31(6), 432-448.

[Patil S, Van Den Berg J, Curtis S, Lin MC and Manocha D., 2010] Directing crowd simulations using navigation fields, IEEE Trans Visualization Comput Graphics 2010; 17(2): 244–254.

[Treuille A, Cooper S and Popovic Z., 2006] Continuum crowds. ACM Trans Graphics 2006; 25(3):1160–1168.

[Teknomo K. 2008., 2008] Modeling mobile traffic agents on network simulation. In: Proceedings of the 16th annual conference of the Transportation Science Society of the Philippines (TSSP), Manila, 2008.

[Teknomo K, Bauer D and Matyus T., 2008] Pedestrian route choice self-organization. In: Proceedings of the 3rd international symposium of simulation in transportation, Gold Coast, Australia, 2008.

[Wang, W. L., Lo, S. M., Liu, S. B., & Kuang, H. 2014] Microscopic modeling of pedestrian movement behavior: Interacting with visual attractors in the environment. Transportation Research Part C: Emerging Technologies, 44, 21-33.

[Dijkstra, J., Timmermans, H. J., & de Vries, B., 2013]. Activation of shopping pedestrian agents—Empirical estimation results. Applied Spatial Analysis and Policy, 6(4), 255-266.

[Borgers, A., & Timmermans, H., 1986]. A Model of Pedestrian Route Choice and Demand for Retail Facilities within InnerCity Shopping Areas. Geographical analysis, 18(2), 115-128.

[Ali, W., & Moulin, B., 2006]. How artificial intelligent agents do shopping in a virtual mall: A 'believable'and 'usable'multiagent-based simulation of customers' shopping behavior in a mall. In Advances in Artificial Intelligence(pp. 73-85). Springer Berlin Heidelberg.

[Werberich, B. R., Pretto, C. O., & Cybis, H. B. B., 2014] Pedestrian route choice model based on friction forces. Simulation, 0037549714547295.

[Helbing D, Farkas I and Vicsek T., 2000] Simulating dynamical features of escape panic. Nature 2000; 407: 487–490.

[Dijkstra E., 1959] A note on two problems in connection with graphs. Numer Math 1959; 1: 269–271.

[Dijkstra, J., Timmermans, H., Jessurun, J., & de Vries, B., 2014]. Modeling Time Duration of Planned and Unplanned Store Visits in a Multi-Agent Simulation of Pedestrian Activity in City Centers. In Pedestrian and Evacuation Dynamics 2012 (pp. 815-823). Springer International Publishing.

[Oppewal, H., & Timmermans, H., 1999] Modeling consumer perception of public space in shopping centers. Environment and Behavior, 31(1), 45-65.