Jornada de Seguimiento de Proyectos, 2005 Programa Nacional de Tecnologías Informáticas

# Study of the Systems with Uncertainty in Presence of a Fuzzy Similarity TIC2003-04564

J. Recasens<sup>\*</sup> Sec. Matemàtiques i Informàtica ETSAV Universitat Politècnica de Catalunya

#### Abstract

In real problems, there are basically two kind of uncertainty: uncertainty due to the lack of knowledge with respect to well defined objects, but for which there are different hypothetical situations, and uncertainty due to the intrinsic vagueness of the objects. Traditionally, the first type of uncertainty has been studied with the help of Probability Theory while the second one has used Fuzzy Set Theory and Possibility Theory. In situations where the two kind of uncertainty are present, theories capable of handling the probability and the possibility of fuzzy events together are needed. On the other hand, many times the vagueness of the objects is due to the existence of a fuzzy equality that makes indistinguishable different objects of the universe. This can happen, for example, because of the imprecision of the measurement tools. Indistinghishability operators or similarities model this kind of equalities in a satisfactory way.

The main target of this project is to formulate a general framework that allows us to deal with different types of uncertainty in universes where a similarity between their objects is defined in a coherent way. This general target can be divided in the following three partial ones:

) Stada a familia di stalia di

a) Study of probability distributions compatible with a given similarity relation.

b) Generation of similarity relations on a universe starting from a similarity defined on a set of prototypes of this universe.

c) Extending the way normalized fuzzy sets generate possibility distributions to the non-normalized case. **Keywords**: Fuzzy relation, Similarity, Uncertainty, Fuzzy sets, Random sets, Possibility Theory, Probability Theory.

## 1 Objectives of the project

For many years, members of our department belonging to this research project have focused their research on Fuzzy logic, especially fuzzy relations and indistinguishability operators. After a long period of theoretical studies and results, there have been some attempts to apply them to a number of branches of AI such as Approximate Reasoning or Data Mining obtaining interesting results that have encouraged the team to find new applications of these relations and to generate new models that allow the treatment of real situations where there are different kinds of uncertainty. Since many times part of the uncertainty on a system is due to the existence of a vague equality that

<sup>\*</sup>Email:j.recasens@upc.edu

### TIC2003-04564

does not allow to distinguish clearly some elements of the universe of discourse, indistinguishability operators seem a good tool to study these systems. Hence, the main goal of this project is the study of systems in which there are different types of uncertainty and to define a general framework for their study.

The project has been divided in three realistic subprojects with the idea of spending roughly a year to each one of them:

1. Study of probability distributions compatible with an indistinguishability operator.

1.1. The study of compatibility relations between a probability distribution and an indistinguishability operator defined on the same universe of discourse.

1.2. Generating probability distributions starting from an indistinguishability operator in a coherent way both from the theoretical and applied points of view.

1.3. Some toy examples.

2. Generation of an indistinguishability operator on a universe starting from an indistinguishability operator defined on a set of prototypes.

2.1. Study of the idea of prototype.

2.2. Searching and characterization of Fuzzy sets that can be considered as prototypes of a concept when there is an indistinguishability operator on a universe.

2.3. Finding methods to generate an indistinguishability operator on a universe when there is a family of prototypes with an indistinguishability operator on it and some matching degrees between the elements of the universe and the prototypes modelled by fuzzy subsets.

2.4. Applicability to real problems.

3. Generalization of the possibility measures to non-normalized fuzzy sets.

3.1. Relation between non-normalized fuzzy sets and (normalized) possibility distributions.

3.2. Generation of normalized fuzzy sets and possibility distributions from non-normalized fuzzy sets.

3.3. Normalizations of belief measures when the empty set has non-zero mass assignment.

3.4. Generation of indistinguishability operators form belief and plausibility functions.

### 2 Success of the project

The main goals for the two first years were

### TIC2003-04564

a) The study of the probability distributions compatible with an indistinguishability operator.

b) Generation of an indistinguishability operator on a universe starting from an indistinguishability operator defined on a set of prototypes.

There have been advances in both directions:

The study of indistinguishability operators in relation with the Theory of Evidence of Dempster-Shafer has allowed us to define indistinguishability operators from belief and plausibility measures and in particular from probability and possibility distributions.

A new way to normalize probability distributions obtained by combining two sources of evidence has been developed. Namely, finding the normalized probability distribution most similar to the obtained one. The results have allowed us to justify a number of well known methods like Demster's rule or based on the cross entropy and to find new ones.

Starting on from prototypes, we have found indistinguishability operators that take the algebraic and the ordering structure of the universe of discourse into account. This is especially interesting in applications where the universes are intervals of the real line or Cartesian products of them.

Another promising point consisting of the generation of aggregation operators from indistinguishability operators has been developed. The basic idea is that in many situations if we want to aggregate the objects a and b we are looking for another object as similar to a than to b, or in a more logical language, with the same degree of equivalence or equality to a than to b.

A more theoretical task has been the study of the structure of indistinguishability operators from their isometry group.

### **3** Indicators

There have been published three journal articles, 9 conference papers and three more articles are being reviewed. They are referenced bellow.

The team of the project has been in touch with the AI Group of the University of Bristol. Dr Jonathan Lawry, a member of this group, has been with us in a sabbatical for three months last year.

Articles:

Demirci, M., Recasens, J. Fuzzy Groups, Fuzzy Functions and Fuzzy Equivalence Relations. Fuzzy Sets and Systems 144 (2004) 441-458.

Jacas, J., Recasens, J. The Group of Isometries of an Indistinguishability Operator. Fuzzy Sets and Systems 146 (2004) 27-41.

### TIC2003-04564

Hernández, E., Recasens, J. Indistinguishability Relations in Dempster-Shafer Theory of Evidence. International Journal of Approximate Reasoning 37 (2004) 145-187.

Lawry, J., Recasens, J., González Rodríguez, I. A Similarity Approach to Evidence Combination in Dempster-Shafer Theory. Sent to Artificial Intelligence.

Jacas, J., Recasens, J. Aggregation Operators based on Indistinguishability Operators. Sent to Int. Journal of Intelligent Systems.

Boixader, D., Recasens, J. A Map Characterizing the Fuzzy Points and Columns of a T-indistinguishability operator. Sent to International Journal of Intelligent Systems

#### Conferences

Jacas, J., Recasens, J. Aggregation of T-transitive Relations using OWA Operators. Proceedings IFSA 2003 Congress. Istanbul. (2003)

Jacas, J., Recasens, J. Aggregating non-finite families of t-transitive relations Proceedings EUSFLAT 2003 Congress. Zittau. (2003). 429-432.

Jacas, J., Recasens, J. Indistinguishability Operators Generated by Prototypes. IPMU2004. Perugia. (2004).

Jacas, J., Recasens, J. Indistinguishability Operators Generated by Fuzzy Numbers. FUZZ'IEEE2004. Budapest. (2004).

Jacas, J., Recasens, J. Preórdenes Duales, Operadores de Clausura y Contextos Borrosos. ESTYLF2004. Jaén. (2004).

Jacas, J., Recasens, J. A Similarity-based Approach to Aggregation. FUZZ'IEEE2005. Reno. (2005).

Tomás, María Santos, Pseudometrics and 3-positive semidefinite similarities, 43 ISFE, Nantes. (2005)

Jacas, J., Recasens, J. Aggregating with Indistinguishability operators. EUSFLAT2005. Barcelona. (2005).

Hernández, E., J., Recasens, J. Indistinguishability in Cooperative Games. EUSFLAT2005. Barcelona. (2005).