DELIA: Deduction and programming in non-classical logics for knowledge technology
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Summary

This project deals with the study of mathematical foundations, as well as deduction and programming, for several types of non-classical logics, especially multiple-valued logics and intermediate logics, with the aim of providing means for intelligent management of information. The classical problem of deduction is, given explicit knowledge expressed in a given formalism together with a set of inference rules, to deduce the implicit knowledge which could be relevant for applications. Specifically, we are interested in those knowledge bases containing uncertain or incomplete information, to develop mechanisms of analysis in which vague or imprecise queries are permitted. A first formal framework chosen is fuzzy logic and different paradigms of fuzzy logic programming. In this project we propose the development of a theoretical framework which generalises different existent approaches to logic programming in vague or imprecise contexts by considering different lattice-like generalised structures as the set of truth-values: bilattices, trilattices, multi-lattices, biresiduated lattices, etc. For this general goal, two additional specific problems will be addressed: firstly, unification in these general contexts, in which categorical techniques are expected to be useful and, secondly, automated deduction in these systems, in which we are planning to use the TAS methodology, which is mechanised deduction framework developed by the Málaga partner which integrates (some of) the benefits of tableaux and resolution; for instance, non-clausal formulas can be analysed directly whereas unit-resolution and the pure literal rule are built-in. From the programming point of view, extensions are sought of the paradigm of Answer Set Programming (ASP) which in turn is already a generalisation of classical logic programming. These extensions of ASP can be implemented either in existing implementations of ASP, or by using systems based on quantified Boolean formulas (e.g. QUIP), or by using TAS. The importance of basic research in these logical systems for Knowledge Technology is driven by the numerous potential applications they offer for modelling and solving problems in areas such as information management, cognitive robotics, Semantic Web, etc.

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1 Objectives of the project

The aim of the first sub-project is to develop deduction methods for lattice-valued logics with applications to soft-computing. Specifically, a general mathematical framework for logic programming extended to a fuzzy context is envisaged which embeds the particular features of the residuated and multi-adjoint approaches to logic programming. The main research objectives in this sub-project are the following:

- Investigate many-valued and fuzzy logics over lattices and other generalised structures: Description of logics taking values in different ordered structures, studying the particularities of their semantics and proof theory.
  Development of a theory of multi-lattices, analysing its algebraic features, including the generalisation of the notions of filter and ideal. Investigate the possibility of extending existing results to these structures.
- Extension of fuzzy logic programming paradigms: Generalisation of existing frameworks by using lattice-valued logics.
  Development of algorithms for (categorical) unification useful for the extended paradigms introduced so far.
- Generalisation of TAS. Development of the mathematical structures needed in order to study the structure of the set of literals of particular logics of interest. Viability of the extension of the concept of \( \Delta \)-tree or alternative data structures for these logics.

The aim of the second sub-project is the study of new or extended systems of non-monotonic reasoning by using non-classical logics, investigate their meta-theoretical properties and their potential applications to practical problems of reasoning in knowledge technologies. The systems to be studied can be viewed as belonging to the new paradigm of answer set programming (ASP) that is currently being used in AI application areas such as planning, diagnosis, information integration and the Semantic Web. More specifically, areas of investigation include:

- Extending the paradigm of answer set programming to deal with aspects such as growth of the knowledge base (induction, abduction, explanations), change in the knowledge base (belief revision, update theory) and approximate reasoning (probabilistic, fuzzy, bayesian).
- Investigate extensions of the language of ASP/equilibrium logic to include modal, causal/action operators and first-order formalisms.
- Study deduction methods and implementation techniques. Investigate potential applications of the resulting systems.

Research tasks

In this section we briefly describe the different tasks scheduled in order to fulfil the objectives stated above:

1. Investigation of many-valued logics over generalised ordered structures:
(a) **Multi-lattices**: Our starting point is the generalisation of lattices in which the concept of supremum (resp. infimum) is substituted by minimal (resp. maximal) of the set of upper bounds (resp. lower bounds), which leads us to work with non-deterministic instead of deterministic operators.

(b) **Bi-lattices, tri-lattices and generalisations.** We study the theoretical and practical implications of working with more than one lattice structure on the set of truth-values with respect to the associated operational semantics. In this direction, it would be worth considering a further generalisation by using an underlying multi-lattice.

(c) We study the particularities of the different many-valued logics obtained when using the type of ordered structures described above as underlying set of truth-values.

2. **Extension of fuzzy logic programming paradigms.**

A framework which increases expressivity is sought. The different possibilities concerning orderings in the underlying set of truth-values should be evaluated regarding their scope and limits. Moreover, the corresponding extension to first-order requires studying the problem of unification in non-canonical contexts. More specific tasks are the following:

(a) **Extending existing fuzzy logic programming paradigms.** Investigate the continuity of the fixed-point operators for the proposed logics. Investigate the existence of sound and complete operational semantics for the proposed systems.

(b) **Developing categorical unification algorithms** to cope with the extension to first-order of the proposed systems. In a fuzzy context, neither the usual unification algorithm nor those based on similarity relations are able to deal with all the extra expressive power of these logics. We believe that an abstract approach to unification at a categorical level could be an interesting solution.

3. **Mechanisation of reasoning.**

(a) **Development of the mathematical structures** needed in order to study the structure of the set of literals of particular logics of interest. Viability of the extension of the concept of $\Delta$-tree or alternative data structures for these logics.

(b) **Definition of a TAS prover for the proposed logics**: Study specific simplification rules, both equivalence- and satisfiability-preserving, for the proposed systems.

4. **Extensions of the current version of equilibrium logic.**

(a) **Generalisations of the concept of strong equivalence and their applications to logic programming.** Equilibrium logic is based on the nonclassical logic of here-and-there. Since this logic characterises the strong equivalence of programs in ASP (strongly equivalent programs are interchangeable in any context), it follows that any transformation or simplification which preserves here-and-there equivalence can be applied to programs as a pre-processing step. An important area of study is to consider whether some types of transformations suggested by results on strong equivalence can provide enhancements to ASP solvers such as $d$lv.
(b) **Extensions of the language.** In logic programming, an important class of extended reasoning systems can be obtained by the inclusion of new operators in the basic logic language, for instance, modal, causal or action operators. Existing constructions are defined in the context of stable models, therefore, it is natural to investigate extensions to nested programs and to equilibrium logic as a whole.

(c) **Abduction and induction.** In previous work by members of the project, equilibrium logic for abductive reasoning was considered. It is planned to continue this work by looking also at the concept of explanation. There are close connections between abductive logic programming and inductive logic programming (ILP). Another task in this context is the development of an ILP system for answer set programming with the ability to deal with generalisations such as nested programs.

(d) **Belief revision and updates.**

In the language of equilibrium logic and extended logic programming, the concept of strong negation is closely related to the idea of incredulity. Previous works showed some relation between the standard methods of belief revision and incredulity. The project will study how belief revision operators can be defined for languages with strong negation and for formalisms such as equilibrium logic.

5. **New versions of equilibrium logic with different underlying logics.**

(a) Extend the equilibrium construction to other multiple-valued logics (either finite- or infinite-valued) and study the behaviour of the logics obtained.

(b) Capturing other LP semantics by means of variants of equilibrium logic. The equilibrium construction in the logic of here-and-there captures reasoning with stable models or answer sets. By modifying the construction (resp. the underlying logic) one may consider whether other logic programming semantics (eg. well-founded semantics) can also be characterised.

6. **Implementation and related tasks.**

(a) Search for a polynomial translation for the complete equilibrium logic into disjunctive logic programs, and implement it as a front-end of d1v. Compare the resulting system with the current version of equilibrium reasoning as implemented in QUIP.

(b) TAS directly implements proof procedures for multiple-valued logics and their extensions, instead of translating a problem in non-classical logic into a problem in classical logic, and then apply ASP or QBF solvers. A first TAS solver for equilibrium logic is being developed and can be used as a starting point for the implementation of a mechanised reasoning system comparable to the methods mentioned above.
Gantt chart

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2 Level of success achieved to date

Most of the proposed tasks are being fulfilled as expected. In the rest of the section we will describe the major achievements obtained in each of the tasks.

The publications [4, 5, 18] are specifically oriented to tasks (1a) and (1c) with marginal contribution to task (1b). In particular, in [4, 5] the structures called multi-semilattice, multi-lattice, universal multi-semilattice and universal multi-lattice are introduced, algebraic characterisations are proposed for them, and their mathematical properties are studied. On the other hand, [18] focuses on a prospective study of the use of the structure of ordered multi-lattice as underlying set of truth-values for a generalized framework of logic programming is presented. Specifically, we investigate the possibility of using multilattice-valued interpretations of logic programs and the theoretical problems that this generates with regard to its fixed point semantics.

Task (2a) is covered by publications [12, 13, 14, 15]. In [14] a neural net based implementation of propositional [0,1]-valued multi-adjoint logic programming is presented. The implementation needs some preprocessing of the initial program to transform it into a homogeneous program; then, transformation rules carry programs into neural networks, where truth-values of rules relate to output of neurons, truth-values of facts represent input, and network functions are determined by a set of general operators; the output of the net being the values of propositional variables under its minimal model. Later, in [16, 12], a generalization of the homogenization process needed for the neural implementation is presented in order to deal with a more general family of adjoint pairs, but maintaining the advantage of the existing implementation. A modification of the neural implementation is presented in [15] in order to deal with a more general family of adjoint pairs, including conjunctors constructed as an ordinal sum of a finite family of basic conjunctors. This enhancement greatly expands the scope of the initial approach, since every t-norm (the type of conjunctor generally used in applications) can be expressed as an ordinal sum of product, Gődel and Łukasiewicz conjunctors. Finally, a different underlying lattice is used in [13], where we pursue an extension of the previous
approaches in order to accommodate calculation with truth-intervals.

The modal logic of qualitative order-of-magnitude reasoning $L(MQ)$ [1] arose as an interesting by-product when studying different approaches to reasoning under imperfect information. Current joint work is being done together with Prof. Ewa Orłowska in order to obtain a Rasiowa-Sikorski proof procedure for $L(MQ)$.

A more theoretical study of properties related to task (2a) has been published in [7, 9, 10], where termination conditions for various extensions of fuzzy logic programming frameworks were introduced.

In [19] a formal model for similarity-based fuzzy unification in multi-adjoint logic programs was introduced: on the multi-adjoint computational model, a similarity-based unification approach is constructed by simply adding axioms of fuzzy similarities and using classical crisp unification which provides a semantic framework for logic programming with different notions of similarity.

Reference [11] introduces advances towards the generalised categorical unification algorithm: a categorical approach involving set and term functors as monads allows for a study of monad compositions that provide variable substitutions and compositions thereof. In this paper, substitutions and unifiers appear as constructs in Kleisli categories related to particular composed powerset term monads. Specifically, we show that a frequently used similarity-based approach to fuzzy unification is compatible with the categorical approach, and can be adequately extended in this setting.

The references [37, 22, 8] contain different approaches to the mechanisation of reasoning and, therefore, are related to task (3b); the first reference introduces a tableaux-inspired prover for equilibrium logic; the second presents a new implementation of the TAS prover for classical propositional logic; the third one introduces an tabulation proof procedure for residuated logic programming, as a first attempt to obtain a such a proof procedure for full multi-adjoint logic programming.

Regarding the second sub-project, its main effort during this period was directed at the following:

- Consolidation and extension of concepts of equivalence between programs or theories in answer set programming and equilibrium logic.

The importance of this topic is evident from the fact that LPNMR (the most important conference on the topic of logic programming and non-monotonic reasoning) dedicated in its 2004 edition a complete session of four talks to this specific theme. In addition, INFOMIX (currently the most ambitious European project on the application of ASP, see http://sv.mat.unical.it/infomix/) initiated the study of variations of this concept for program optimisation for applications in the field of data integration.

Reference [26] proposes a characterisation of uniform equivalence in equilibrium logic and a tableaux system for checking uniform equivalence, a first implementation of which has been developed and described in [37]. A second paper, [27], is concerned to a more significant extension of the concept of equivalence: the synonymy between theories formulated in different languages. Works [21, 2] study strong equivalence relative to the semantics of paraconsistent answer sets (PAS) and propose a new representation of models. Regarding the optimisation of computational systems, an application of strong equivalence to the
transformation and simplification of programs can be seen in [23]. These transformations could be implemented as soon as we have efficient systems of automated deduction for the here-and-there logic.

- Extension of the language of equilibrium logic.

The project foresees a series of possible extensions of the language of equilibrium logic, such as the addition of modal or causal operators, in order to extend the current languages of action based on ASP. In this first period of the project we have focussed on first-order extensions since, in the short term, they seem to have greater potential impact on extensions of existing computational systems. (However a partial, modal extension of equilibrium logic has recently been studied by Wand and Zhang (LPNMR 05).) The first results obtained [28, 25] are promising in that we already have a first-order version for the logic N5 with constant domains for here-and-there models with nice meta-theoretical properties and capturing the inference relation of ASP for programs with variables.

The future research line on the integration of ASP and multi-agent systems has two main goals. The first one is the extension of agent programming languages based on ASP, taking into account the organising context provided by the multi-agent systems in which they are embedded. A preliminary work in this direction (although without mentioning ASP) is [34]. The second goal deals with the specification of protocols of interaction by means of ASP, starting from the conceptual framework established in [33].

- Other variants of equilibrium logic.

Two generalisations have been studied recently. In [20] it is shown how a paraconsistent version of the logic N5 can be used to capture paraconsistent answer sets. In a further paper under submission it is shown how a partial version of equilibrium logic (based on a previously unknown nonclassical logic) captures reasoning with well-founded semantics.

- Contribution to the development of computational systems.

This theme is covered by references [26] (tableaux system), [23] (transformations) and [2], which introduces two types of reduction of propositional theories in equilibrium logic to logic programs. The second method described in [2] seems to be adequate as a basis for the implementation of equilibrium logic as a front-end for dlv.

Last but not least, it is worth mentioning the automated system described in [37] for equilibrium logic, which includes both a strong equivalence and a uniform equivalence checker.

- Foundations of fuzzy logic and reasoning

The Madrid partner has also been an active contributor to the area of fuzzy logic, treated mainly in subproject 1. In [35, 36] distinct properties of fuzzy inference are studied, in particular verification of the meta rules of modus ponens/modus tollens and of contrasymmetry for the case of fuzzy implication. The papers [3, 31, 32] deal with the theory of aggregation. [3] proposes a new mechanism for aggregation that takes account of the origin of the information to be aggregated, while [31, 32] deal with the principles of non-contradiction and excluded middle.
3 Progress indicators

After fifteen months since his incorporation to the project, the PhD student associated to the first sub-project has begun to actively collaborate in some specific research tasks. As a result, a workshop contribution has been already accepted and a draft paper has been submitted for publication.

Members of the research team have been invited to give a talk or organise a special session in different international conferences:

- In July 2004, A. Valverde and D. Pearce were invited to the workshop “Negation in Constructive Logic” held in Dresden (Germany). During this workshop, they contacted other researchers whose topics are related to the goals of this project, in particular with Sergei Odintsov from the Sobolev Institute of Mathematics (Russia).

- Simultaneously, J. Medina and M. Ojeda presented an invited talk on a special session at IPMU dedicated to reasoning under imperfect information [17] surveying on recent and prospective results about multi-adjoint logic programming. Later, when attending the ECAI and JELIA conferences, M. Ojeda initiated a collaboration with Umberto Straccia, from CNR (Italy), and Carlos Damásio, from Univ. Nova de Lisboa, in order to develop the mathematical foundations of a further extension of logic programming to manage uncertainty. This collaboration resulted in the application of a proposal of Integrated Action between Spain and Italy for the period 2006–2007.

- In April 2005, A. Valverde and D. Pearce participated, by invitation, in the seminar “Nonmonotonic Reasoning, Answer Set Programming and Constraints” held in Dagstuhl (Germany), where prospective collaboration with other research groups, specially, from the European project WASP was discussed.

- Moreover, a special invited session in EUSFLAT will be organised by J. Medina and M. Ojeda on the topic of Non-Classical Unification and Generalised Logic Models.

Regarding publications, here we present a summary of journal publications, peer-reviewed national/international conference proceedings and workshop contributions.


**Natl. Conf.** ESTYLF: Spanish Conference on Fuzzy Logic and Technologies [9, 16], LFSC: Lógica Fuzzy & Soft Computing [18].

Last but not least, there is a substantial level of collaboration with other groups, as shown:

1. with Pedro Cabalar of the University of Corunna on the foundations of nonmonotonic reasoning. Prof. Cabalar has joined the project as an associate to the Madrid partner.

2. with Carlos V. Damásio from Univ. Nova de Lisboa, on the topic of termination results and tabulation procedures for different extensions of fuzzy logic programming.

3. with Thomas Eiter, Hans Tompits and Stefan Woltran of the Technical University Vienna on implementing equilibrium logic and on variations of strong equivalence.

4. with Patrik Eklund and Mª Ángeles Galán from Umeå University, on the generalised categorical unification algorithm. Dr. Galán is joining the Málaga partner as a Juan de la Cierva Research Fellow (2005).

5. with Sergei Odintsov of the Sobolev Mathematical Institute Novosibirsk on logical systems with nonclassical negation.

6. with Ewa Orłowska from the National Telecommunications Institute in Warsaw, on Rasiowa-Sikorski proof methods for the logic of qualitative order-of-magnitude reasoning.

7. with Axel Polleres of the University of Innsbruck on answer set programming and the semantic web. Dr. Polleres is joining the Madrid partner as a Juan de la Cierva Research Fellow (2005).

8. with Umberto Straccia from Consiglio Nazionale delle Ricerche (Italy) about mathematical foundations of a further extension of logic programming to manage uncertainty.

9. with Peter Vojtáš from Charles University of Prague, on the development of the theory of multi-adjoint logic programming.
References


