

jspTIN-2007: Multi-valued Logic: Foundations and application to management of vagueness and imprecision TIN2004-07933-C03

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Abstract

The project had three basic objectives. The first one was the logical and algebraic study of t-norm based many-valued logics in the frame of both residuated and substructural logics. The second one was the formalization, within the framework of the above logics, of several deductive soft computing mechanisms, based on fuzzy logic, to deal with vagueness, uncertainty and imprecision. The last one was to continue theoretical and experimental research on many-valued SAT and Max-SAT algorithms and their application to solve computationally difficult problems. We have deeply worked on the three goals and obtained a significant number of results, which we summarize in this report and which have been published in several conferences and journal papers.

Keywords: Multivalued Logics, t-norm based Logics, Substructural Logics, Multivalued SAT and Max-SAT, Soft Constraints

1 Project goals

The objective of the project is the study of multi-valued logics and their computational applications. Mainly we have proposed to study both infinitely-valued ones (fuzzy logics valued over $[0,1]$) and their algebraic semantics and finitely-valued ones typically used in SAT problems. The basic assumption of the first goals of the proposal was that residuated many-valued (fuzzy) logics, (what was known as t-norm based -fuzzy- logics) are a good tool to formalize vagueness and imprecision. From this assumption, the project goals described in the proposal are the following ones:

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- 1 .- *Logical and algebraic study of t-norm based logics.* MULOLOG project proposes the study of quasi-varieties of BL and varieties of MTL and IMTL.
- 2 .- *Study of t-norm based logics from the substructural point of view.* Being t-norm based logics nice and interesting examples of logics without contraction, MULOLOG project proposes to study these logics from the point of view of substructural logics, mainly starting from the monoidal structure of the corresponding algebras. In the proposal we say that perhaps their study would need more than Hilbert and Gentzen formalisms (axiomatic and Gentzen systems) and we would need to use Avron's formalism (mainly hypersequents).
- 3 .- *Free algebras and functional representation of substructural and t-norm based logics.* The goal was to study free algebras in order to describe the structure of the algebras of propositional formulas. Moreover we are interested in functional completeness theorems like Mc Naughton for the infinite valued Lukasiewicz logic.
- 4 .- *Formalization of some Soft Computing techniques in the framework of t-norm based logics.* Here the proposed goals were:
 - i) To study the logic obtained by adding truth constants to the logic of a t-norm and its residuum (specially in the case of a continuous t-norm), with special attention to evaluated formulas, the kind formulas used in many fuzzy knowledge-based systems or in fuzzy logic programming.
 - ii) Similarity based reasoning. Special attention is paid to both coherence problems and interpolation in fuzzy rule based systems.
 - iii) To model Soft-CSP problems using the language of the extended possibilistic logic with fuzzy constants.

The basic assumption of the last two goals is that many-valued clausal forms provide a suitable formalism for representing and solving decision problems via their reduction to SAT and optimization problems via their reduction to Max-SAT:

- 5 .- *Design, implementation and evaluation of SAT algorithms for multivalued clausal forms and their application to solve computationally difficult problems.* The goal was to incorporate in our algorithms both the recent techniques introduced in Boolean SAT, as well as new multivalued branching rules and variable selection heuristics that take into account the structural information provided by the variable domains.
- 6 .- *Design, implementation and evaluation of Max-SAT algorithms for multivalued clausal forms and their application to solve problem with soft constraints.* The project proposes to work on this topic, which was unexplored when we wrote the proposal of the project.

Eventhough there is a "crono-gram" in the proposal we think that the work in these general goals is a continuous task during the three years of the project and thus we will present in the next section the results we have achieved at the end of the second year (December 2006).

2 Project results

We next describe the results obtained in the development of the project for each one of the goals described in the previous section.

- 1 - *Logical and algebraic study of t-norm based logics.* The results obtained in this goal are mainly referred to varieties of MTL and IMTL and only someones related to varieties (not yet quasi-varieties) of BL. We have published the two papers [EGM04] and [DEGM05] about subvarieties of BL. The first proves that the logic of a continuous t-norm and its residuum is finitely axiomatizable and gives an algorithm to find a finite system of axioms for each continuous t-norm, result that is used posteriorly in the study of the logic obtaining by adding truth constants (see results of goal 4). The second is a extended survey of known results (and some news ones) about subvarieties of BL in a special issue of a journal edited by two of the members of the IIIA team in the MULOLOG project. In the MTL and IMTL cases we have published the four journal papers [GNE05, NEG05b, NEG05a, Gis03, GT05, MNH06]. This papers describe a large amount of subvarieties of MTL and IMTL with their axiomatizations and standard completeness results. Moreover, they present different methods to try to obtain a complete description of MTL and IMTL subvarieties and even thou this goal is not completely obtained the results are relevant. Moreover we have also presented several communications to conferences (among them we cite [nNE04, NEJ05, NEJ06]) and have been invited to give general talks in the EUSFLAT'06 conference (the conference of the European Society for Fuzzy Logic and Technology) [Est05a] and in the Seminar on Fuzzy Set Theory in Linz [Est05b].
- 2 - *Study of t-norm based logics from the substructural point of view.* The results concerning this section are contained in the papers [VGCB04a, VGCB04b, VGCB06a, VGCB06b, TVFG06]. T-norm based logics are truth-preserving logics and they can also be seen as substructural logics because they are extensions of the Intuitionistic logic without contraction (or monoidal logic) ML. The study of ML is very useful to obtain general properties of t-norm based logics. We have proved in [VGCB06a] that the fragment without implication and without negation is exactly the corresponding fragment of classical logic. As a consequence the same situation happens for every t-norm based logic, in other words when we do not deal with the connectives of implication and negation the behaviour of t-norm based logics is exactly the same as classical logic. In particular for every t-norm based logic, its fusion-fragment and its conjunction-fragment are equal. This kind of pathological behaviour will be analyzed in future papers and is mainly due to the fact that these logics are truth-preserving. In [VGCB04b, VGCB06b] we have studied the fragment without implication and with negation of the substructural logic ML. This fragment is strictly included in the corresponding fragment of Intuitionistic logic (and so in the corresponding fragment of classical logic). In [TVFG06] we have started the study of t-norm based logics preserving degrees of truth. Preserving degrees of truth means that the degree of truth of a consequence of a finite set of formulas is greater or equal than the degree of truth of every one of these formulas. We obtain a Gentzen-style and a Tarski-style axiomatization for the infinite-valued Lukasiewicz that preserves degrees of truth. We hope that we will obtain a Hilbert-style axiomatization for this logic in a near future.
- 3 - *Free algebras and functional representation of substructural and t-norm based logics.* The results obtained in this goal are descriptions of free algebras of varieties of algebraic structures related with MTL-logic. In [TC06] we give a complete description of free algebras in varieties of some special cases of BP_0 -algebras, concretely the MTL-algebras

with Glivenko property and boolean retract. In [TV] we show that, in general, the variety of all Łukasiewicz implication algebras, corresponding with the implicative fragment of the logic given by the Łukasiewicz t-norm, has its free algebras directly indecomposable. In [GT07] we obtain that the free algebra of bounded BCK-algebras, corresponding with the implicational and negational fragment of Monoidal logic, is semi-simple; from this result we also show (with some difficulty) that the variety generated by bounded BCK-algebras is generated by its simple finite members. To obtain the description of the free algebras in varieties of MTL-algebras we use boolean representation, method that cannot be used in Łukasiewicz implication algebras, which need a more direct description. To analyze free bounded BCK-algebras we refer to some results on residuated lattices.

- 4 .- *Formalization of some Soft Computing techniques in the framework of t-norm based logics.* Most relevant results in this goal are devoted to item i), the study of the logics obtained by adding truth constants to the logic of a t-norm and its residuum. We have published three journal papers [SCE⁺06, EGN06, EGGN] and a chapter of a book [EGN]. The two first papers contain full completeness results for Product and Gödel logics with any set of truth constants associated to a countable subalgebra of the corresponding standard algebra. The last journal paper contains general completeness results for any logic of a continuous t-norm and its residuum by adding any set of truth constants associated to a countable subalgebra of the standard algebra. In both papers special attention is paid to the case of evaluated formulas, formulas of type $\bar{r} \rightarrow \varphi$ where \bar{r} is a constant and φ is a formula without constants. Finally the book paper contains a survey of all results plus new completeness results for the logic of the nilpotent minimum t-norm (which is not continuous) and its residuum with truth constants. Moreover we have presented several communications to conferences and workshops, among them we cite [EGN04, CEGN04]) and we note that members of the IIIA team have been invited to give plenary talks in the conferences “The logic of Soft Computing and Workshop of the ERCIM working group on Soft Computing” in 2005 (Ostrava, Czech Republic) [Est05c] and in 2006 (Málaga, Spain) [God06].

With respect to item ii) we have presented the communications [ERBMD04, BMEG⁺05] and the chapter book [AEGT04]. In the communications we have explored new formal tools for interpolation based on similarity and in the book chapter we have given a method to obtain similarities relations in fuzzy case-based reasoning. Moreover we have to say that we have not worked in Soft-CSP mainly because we have taken advantage of the fact that Prof. Chesnevar (from Universidad Nacional del Sur, Bahia Blanca, Argentina) obtained a 5-year contract in the Ramon y Cajal program in Lleida University. Since he is a reputed researcher in argumentation, we have worked in the extension of argumentation systems to deal with possibilistic uncertainty and in this subject we have published several joint papers [CSAG04, CSAG05, CnSG05, CSGA05, ACG⁺06].

We have also published some papers on fuzzy modal formalizations of probability logics [GM06, FG], possibilistic decision theory [GZ05, GZ06] and some on graded BDI architectures for agents [CGS05, CGS06]. These papers and the papers on argumentation have in common the management of uncertainty in the studied systems extending the “classical systems” using mainly classical logic.

Finally we have collaborated in a book chapter [DEGP05] where a general view about

what is vagueness and how to manage it in different scenarios from the point of view of information systems is given. We want to stress the interest of this paper since it has been published in a philosophical oriented book where vagueness is studied from a different point of view and with different formalizations. The discussion there has been very interesting and fruitful.

- 5 - *SAT algorithms for multivalued clausal forms and their application to solve computationally difficult problems.* The results obtained about the SAT problem of many-valued clausal forms have been published in [AM04, AM05a, ABCM04, AdVD⁺04, BMC⁺07]. In [AM04, AM05a] we have defined mappings that translate many-valued clausal forms into Boolean clausal forms, which can be used to solve many-valued SAT instances with Boolean SAT solver as well as to define more efficient Boolean SAT encodings of combinatorial problems with many-valued variables. In [ABCM04] we have given new complexity results for some classes of many-valued SAT problems and analysed experimentally the frontier between P and NP many-valued SAT problems. In [AdVD⁺04] we have defined a variable selection heuristic for solving quasigroup completion problems with many-valued and CSP solvers that outperforms all the existing heuristics. In [BMC⁺07] we have described both complete and local search solvers for the subclass of many-valued clausal forms known as Regular CNF formulas, and proved that encoding combinatorial problems as instances of Regular SAT and solving them with a Regular SAT solver can accelerate the search for a solution. We have also defined a generator of computationally difficult sudokus encoded as SAT instances [ABF⁺06].
- 6 - *Max-SAT algorithms for multivalued clausal forms and their application to solve problem with soft constraints.* The research on many-valued Max-SAT has also produced relevant results on Boolean Max-SAT and Weighted CSP. A complete inference system for Many-Valued Max-SAT and its relationship with the soft local consistencies defined for Weighted CSP is described in [ABLM07], and the first solver we have developed for Many-Valued Max-SAT is detailed in [ADL⁺06]. We have also designed and implemented two solvers for Boolean Max-SAT: MaxSatz and Lazy [AMP04, AMP05]. MaxSatz won the First Max-SAT Evaluation, a co-located event of SAT-2006. It is a branch and bound solver that incorporates the lower bound computation methods defined in [LMP05, LMP06] and refinements of the complete inference resolution rule for Max-SAT defined in [BLM06]. Finally, we have defined a clausal form formalism [AM05b, AM06] and the corresponding Max-SAT solvers for dealing with problems with hard and soft constraints.

3 Quality indicators of the project achievements

In the framework of the project six new researchers have been formed: C. Noguera has obtained his Ph.D. in the University of Barcelona in June 2006 (supervisors: F. Esteva (IIIA) and J. Gispert (UB)), E. Marchioni has obtained his Ph.D. in the University of Salamanca in November 2006 (supervisors: M. Manzano (US) and L. Godo (IIIA)), C. Ansótegui has obtained his Ph.D. in the University of Lleida in July 2004 (supervisor: F. Manyà (UdL)), A. Garcia-Cerdaña will defend his Ph.D. at the beginning of 2007 (supervisors: V. Verdú and F. Esteva), J. Planes will defend his Ph.D. at the beginning of 2007 (supervisors: F. Manyà

(UdL) and Chu-Min Li (Picardie)), and J. Argelich plans to defend his Ph.D. at the end of 2008 (supervisor: F. Manyà (UdL)). Moreover C.Noguera has obtained the students Galois award of the Catalan Mathematical Society in 2005.

A second indicator is the coauthors of papers that is a deep indicator of the relationships that the members of the team have with well recognized foreign researchers. In fact the MULLOG researchers have consolidated relations (many of them supported by bilateral projects¹) with Prof. Hájek, Novak, Cintula and Horcick (Czech Republik (bp)), the Italian researchers Prof. Montagna (Siena) and Prof. Di Nola (Salerno), the Argentinian researchers Prof. Cignoli and R.Rodríguez (Buenos Aires (bp)), A. Casali (Rosario (bp)), G. Simari (Bahia Blanca) and A.Zapico (Cordoba), the French researchers Prof B.Bouchon and M. Rifqi (Paris(bp)), Prof. D.Dubois and Prof. H. Prade (Toulouse(bp)) and Prof. Chu-Min Li (Picardie), Prof. Gottwald (Germany), Prof. M. Baaz and A. Ciabattoni (Austria(bp)), Prof. C. Gomes (USA), as most important.

The third indicator is the invited talks in conference and workshops and the awards both for papers and software already cited in the corresponding sections.

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