

Programación Multiobjetivo: Métodos Metaheurísticos y Aplicaciones TIN2006-13897

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Abstract

The aim of this research, which combines two different fields: Multicriteria Decision Making and Metaheuristic Methods, is the theoretical and applied development of metaheuristic techniques for the resolution of complex Multiobjective Programming problems, that is, those problems whose solutions cannot be achieved by a traditional method. From the theoretical point of view, we will focus firstly on the evolution of a widely used tool for this type of methods, the ϵ -dominance, into a new dominance scheme, the adaptive ϵ -dominance, that maintains the good properties of ϵ -dominance but improves its weaknesses. Secondly, we will focus on the cooperation of this evolved tool and the metaheuristic itself, in such a way that the useful information that can be provided by the adaptive ϵ -dominance, could be used to improve the metaheuristic's performance. To finish the theoretical studies, these methods will be completed with some tools, coming from the field of Interactive Multiobjective Programming, to facilitate their application in real cases. These different methods and tools will be implemented together into a computational package of Metaheuristic Interactive Multiobjective Programming for the resolution of real complex problems. Finally, we want to use this research to solve some complex multicriteria problems, which are being studied currently in our group, within the field of Artificial Intelligence and Environmental Management.

Keywords: Multiobjective Optimization, Metaheuristics, Artificial Intelligence, Environmental Management.

1 Objetivos del proyecto

Below, the main goals (for the three years) of our project are described:

1. Development and implementation of a Multiobjective Metaheuristic, able to efficiently use the information coming from an ϵ -dominance grid. In this goal, we want to study of the combination of an Evolutionary Multiobjective Optimization (EMO) method and a local search method inspired on Rough Sets theory as a viable way of obtaining a good approximation, both in quality and diversity, of the Pareto Set of a multi-objective optimization problem. Our main motivation for such a hybrid approach is to reduce the overall number of fitness function evaluations performed to approximate the Pareto Set of a problem. For this aim, we want to make an EMO method cooperate with the ϵ -dominance grid by using a local search method inspired on rough sets theory, in order to improve the overall performance by sharing more information.

2. Development of an evolution of the ϵ -dominance concept into the adaptive ϵ -dominance tool. Efficiency has become one of the main concerns in EMO during recent years, as one alternative to achieve a faster convergence is to use a relaxed form of Pareto dominance that allows regulating the granularity of the approximation of the Pareto Set to be achieved. One such relaxed forms of Pareto dominance that has become popular in the last few years is ϵ -dominance, which has been mainly used as an archiving strategy in some Multiobjective evolutionary algorithms. Despite its advantages, ϵ -dominance has some limitations, as shown in the literature. In this goal, we want to design a mechanism that can be seen as a variant of ϵ -dominance, which we call adaptive ϵ -dominance, that tries to overcome the main limitations of ϵ -dominance.

3. Design and implementation of an Interactive method for the resolution of a Multiobjective problem using a metaheuristic. Most of the Multiobjective Metaheuristics focus on the approximation of the Pareto Set without including DM's preferences. However, especially in real cases, the determination or approximation of the Pareto Set is not enough, and the decision maker's (DM) preferences have to be incorporated in order to determine the solution that better represents these preferences. But very few works can be found using Multiobjective Metaheuristics and incorporating DM's preferences, and a common fact in all of them is that many modifications on the main architecture have to be done in order to include DM's preferences into the Multiobjective Metaheuristic. In this goal, we want to design an Interactive method that will allow us to include easily the DM's preferences into any Multiobjective Metaheuristic, without having to modify the main architecture of the specific search engine adopted, and based on an interaction frame where the DM is shown a set of representative solutions

4. Integral Forestry Management including aspects related with economical and ecological performance, with application to the area of Pinar del Río (Cuba). With this goal, we want to develop a Multiobjective Programming model to plan the management of a tree plantation, taking economic and environmental objectives into account, while bearing in mind technical restrictions regarding treatments, spatial adjacency constraints, maximum adjacent surface area to which clearcutting can be applied and some more. The proposed model will be applied to a Cuban plantation belonging to the forestry company "Empresa Forestal Integral Macurijé", located in the region of Pinar del Río.

On the other hand, the lack of integration among the factors involved in the wood sawing planning process creates a gap in the production system and, thus, technical-economic inefficiency in these industries. But, the predominant aim of a sawing plan has been to maximize production, although other important aspects are evident, such as performance, production level, warehousing availability and characteristics of raw material, etc. Thus, the use of a Multiobjective model may enrich a sawing plan with all the concurrent criteria for this problem, and this was one of our goals in this project. For this aim, we want to develop a Multiobjective model for sawmill planning, such that the different aims of the decision centre are considered, together with certain constraints regarding availability, and where a Goal Programming approach will be used to include the decision centre's preferences.

5. Optimal design of a Rough Sets grid. The basic ideas of rough sets based on dominance principles have been shown very useful to multicriteria classification problem in the literature. The

problem with this kind of method is the high complexity of the underlying problem to find the optimal grid offering the best performance to the Rough Set method. This grid has a key point on the trade-off between precision and cardinality of the set of rules obtained by the method, composing a bi-objective non-linear combinatorial optimization problem whose resolution cannot be faced with an exact method. This way, in this project we tried to design a Multiobjective Metaheuristic based on Tabu Search for the resolution of this problem. The results obtained in the optimal design of this grid will be used also as a part of the Multiobjective Metaheuristic to be developed also in this project.

In order to attain these goals, this was our proposed chronogram for the first two years:

First year: Development of the adaptive ε -dominance. Applications in Artificial Intelligence and Forestry Management.

Second Year: Development of an Interactive Metaheuristic method for Multiobjective Programming. Applications in Artificial Intelligence and Forestry Management.

2 Nivel de éxito alcanzado en el proyecto

2.1 Development and implementation of a Multiobjective Metaheuristic, able to efficiently use the information coming from an ε -dominance grid.

The aim of this goal (carried out in collaboration with Prof. Coello) is to show how the hybridization of a fast Multiobjective Evolutionary algorithm and a local search method based on the use of rough sets (that we called DEMORS), is an efficient alternative to obtain a robust algorithm able to solve difficult constrained multi-objective optimization problems. The main idea is to use this sort of hybrid approach to approximate the Pareto Set of a constrained multi-objective optimization problem with a low computational cost (only 10,000 fitness function evaluations). The hybrid operates in two stages: in the first one, a multi-objective version of differential evolution is used to generate an initial approximation of the Pareto Set. Then, in the second stage, rough sets theory is used to improve the spread and quality of this initial approximation, using information coming from the ε -dominance grid. We decided to design this approach to solve, not only unconstrained multi-objective optimization problems, but also constrained, taking into account the increase in the demand of multi-objective solvers for real cases, where most of the problems are (hard) constrained. It is worth noting, however, that in spite of this demand, the number of Multiobjective metaheuristics developed with a particular emphasis on reducing the number of objective function evaluations for constrained problems, is very scarce. Taking into account these last facts, we adapted and tested this hybrid method in order to validate it to be used for constrained multi-objective optimization problems.

Differential Evolution (DE) is a relatively recent heuristic designed to optimize problems over continuous domains. DE has been shown to be not only very effective as a global optimizer, but also very robust producing in many cases a minimum variability of results from one run to another. DE has been extended to solve multi-objective problems by several researchers but, in such extensions, DE has been found to be very good at converging close to the true Pareto Set (i.e., for coarse-grained optimization), but not so efficient for actually reaching this set (i.e., for fine-grained

optimization). Thus, we wanted to show how these features can be exploited by our hybrid, which uses rough sets theory as a local optimizer in order to improve the spread and convergence of the efficient solutions obtained by our differential evolution implementation.

On the other hand, Rough Sets theory is a new mathematical approach to imperfect knowledge. It was proposed by Pawlak in 1982, and has been used by many researchers and practitioners all over the world and has been adopted in many interesting applications. Basic ideas of rough set theory and its extensions, as well as many interesting applications, can be found in books, special issues of journals, proceedings of international conferences, and in the internet (see www.roughsets.org). For our case, Multiobjective Optimization (MO) problems, we will try to approximate the Pareto front using a Rough Sets grid. To do this, we will use an initial approximation of the Pareto front (provided by any other method) and will implement a grid in order to get more information about the front that will let us improve this initial approximation. Then, at this point we had to face the following problem: the more precise the grid is, the higher the computational cost required to manage it. Conversely, the less precise the grid is, the less knowledge we get about the Pareto Set. Thus, we needed to design a grid that balances these two aspects, and used for this aim all the research carried out in this field throughout this project. In other words, we had to find a grid that is not so expensive (computationally speaking) but that offers a reasonably good knowledge about the Pareto Set to be used to improve the initial approximation. But, since the computational cost of managing the grid increases with the number of points used to create it, we must try to use just a few points. However, such points must be as well distributed as possible, because the better the distribution the points have in the initial approximation, the less points we need to build a reliable grid. For this aim, we used a Pareto-adaptive ϵ -dominance tool that was used to select an initial set of efficient points for the Rough Sets tool with the best trade-off between cardinality and distribution. This is, the Pareto-adaptive ϵ -dominance grid played the role of communicating both phases, the Differential Evolution and the Rough Sets tool.

In order to validate our proposed approach, our results are compared with respect to those generated by the NSGA-II, which is the most representative method of the state-of-the-art in the area. Both methods were used to solve 7 test constrained problems from the literature, selected trying to cover all different complexities: convex, non-convex and disconnected Pareto Sets, linear and nonlinear objective functions and constraints or the number of decision variables. And both methods were limited to consume only 10,000 fitness function evaluations, which is sensibly less than the standard setting for this kind of problems (around 100,000). Our results indicated that DEMORS outperformed NSGA-II and, thus, is a competitive method for constrained multi-objective optimization problems when performing only 10,000 fitness function evaluations, and it is also able to ensure a good spread and distribution of the solution within this reduced number of evaluations. Experiments were completed with a comparison between first and second phase, in order to ensure that both phases are needed, and a real case related to the Mexican Economy, where the effects of the publish investment in the social and economical development in Mexico are studied. This led us to conclude that the hybridization of a fast evolutionary method with a local search is a suitable tool. This research was the core of the Ph.D. dissertation of Luis V. Santana, one of the students of Prof. Coello, and produced these publications: [3], [5], [8] and [9].

2.2 Development of an evolution of the ϵ -dominance concept into the adaptive ϵ -dominance tool. This research has been carried out within the collaboration with Prof. Coello, and its main goal was the development of an evolution of the ϵ -dominance

concept. Laumanns proposed in 2002 a relaxed form of dominance for multi-objective evolutionary algorithms, named ϵ -dominance, that has been widely used in the literature. This mechanism acts as an archiving strategy to ensure both properties of convergence towards the Pareto Set and properties of diversity among the solutions found. With the ϵ -dominance, a point $\mathbf{f} \in \mathbf{R}^m$ not only dominates those points with lower or equal fitness in all their objectives and strictly lower in at least one objective, but also all points close enough to \mathbf{f} (i.e., those with a distance to \mathbf{f} less than an ϵ). This value, ϵ , can be provided by the DM to control the size of the solution set. Nevertheless, because the geometrical characteristics of the Pareto Set (concavity, convexity, curvature, torsion, disconnected segments, etc.) are usually unknown to the DM, we can lose a high number of good solutions if the ϵ value is badly chosen. Another important limitation of this mechanism is the fact that it loses solutions lying on segments of the Pareto front that are almost horizontal or almost vertical, as well as the extreme points of the Pareto front. This has a direct impact on the spread of solutions along the Pareto Set.

These drawbacks produce the following limitations of the ϵ -dominance tool:

1. We can lose a high number of efficient solutions if the DM does not take into account (or does not know beforehand) the geometrical characteristics of the true Pareto front of the problem to be solved.
2. It is normally the case that we lose the extreme points of the Pareto front, as well as points located in segments of the Pareto front that are almost horizontal or vertical, as shown in Figure 1.
3. The upper bound for the number of points allowed by a grid is not easy to achieve. For a non-adaptive grid, the upper bound is only achieved when the real Pareto front is linear.

We have provided an extension of ϵ -dominance that shares its good convergence properties while addressing the problems indicated above. Our proposal is called Pareto-adaptive ϵ -dominance (pa ϵ -dominance), we consider not only a different ϵ value for each objective but also the vector $\boldsymbol{\epsilon}=(\epsilon_1, \epsilon_2, \dots, \epsilon_m)$ associated to each $\mathbf{f} = (f_1, f_2, \dots, f_m)$ depending on the geometrical characteristics of the Pareto Set. In other words, we consider different intensities of dominance for each objective according to the position of each point along the Pareto Set. Then, the size of the boxes will be adapted depending on the area in the objective functions space so that boxes will be smaller where needed (normally at the extremes of the Pareto Set), and larger in other less problematic parts of the front. In order to validate our proposed pa ϵ -dominance, we adopted three algorithms: Two of them use ϵ -dominance, and in one of them, this type of mechanism is replaced by our pa ϵ -dominance to make the third algorithm. This allowed us to show also the performance of the same algorithm with and without pa ϵ -dominance. We solved five continuous (unconstrained) test problems with different geometrical characteristics for our experimental study. Note that our choice of problems was directed by the geometrical characteristics of the Pareto fronts rather than by the difficulty of solving each test problem, since our goal is to show the advantages of our pa ϵ -dominance scheme over the original ϵ -dominance. The main goal of pa ϵ -dominance is to obtain as many Pareto optimal solutions as possible (up to the maximum capacity of the grid), but within a homogeneous spread. Thus, the performance measures adopted in our study were focused on aspects related not only with the cardinality of the approximation found, but also on its

distribution, as the Chi-Square-Like Deviation measure, the Spread measure and the standard deviation of the Crowding distances, and with all of them adaptive dominance outperformed standard dominance. These experiments led us conclude that $\text{pa}\epsilon$ -dominance is an advantageous alternative to ϵ -dominance especially for those problems with special geometrical characteristics. This research was published in [4].

2.3 Design and implementation of an Interactive method for the resolution of a Multiobjective problem using a metaheuristic, based on an interaction frame where the DM is shown a set of representative solutions. This research has been carried out in two branches: first one with Prof. Coello, focused on the g-dominance concept, and a second one with Prof. Kaisa Miettinen, one of the most recognized researchers in the field of Interactive MO methods, focused on the concept of box indices.

2.3.3 g-dominance. One of the main tools for including DM preferences in the Multiobjective literature is the use of reference points and achievement scalarizing functions. The core idea in these approaches is converting the original problem into a single-objective optimization problem through the use of a scalarizing function based on a reference point. As a result, a single efficient point adapted to the DM's preferences is obtained. However, a single solution can be less interesting than an approximation of the Pareto Set around this area. We designed a variation of the concept of Pareto dominance, called g-dominance, which is based on the information included in a reference point and designed to be used with any EMO method or any MO metaheuristic. This concept will let us approximate the Pareto Set around the area of the most preferred point without using any scalarizing function. On the other hand, we showed how it can be easily used with any EMO or any MO metaheuristic (just changing the dominance concept) and, to exemplify its use, we showed some results with some state-of-the-art-methods and some test problems. This research was published in [7].

2.3.4 Box Indices. Due to the difficulties of dealing with multiple objectives, the way information is presented plays a very important role. Questions posed to the DM must be simple enough and information shown must be easy to understand. For this purpose, visualization and graphical representations can be useful and constitute one of the main tools used in the literature. In this work, we proposed to use box indices to represent information related to different solution alternatives of MO problems involving at least three objectives. Box indices are an intelligible and easy to handle way to represent data. They are based on evaluating the solutions in a natural and rough enough scale in order to let the DM easily recognize the main characteristics of a solution at a glance and to facilitate comparison of two or more solutions in an easily understandable way. This research was published in [6].

2.4 Integral Forestry Management including aspects related with economical and ecological performance, with application to the area of Pinar del Río (Cuba). With this goal, we wanted to develop a Multiobjective Programming model to plan the management of a tree plantation, taking economic and environmental objectives into account. The aims are to stay within the limits of a given harvesting volume, limit the age of basic units targeted for clearcutting, obtain a forest with a balanced age distribution, and surpass the minimum net present value set at each planning period. This has to be achieved bearing in mind technical restrictions regarding treatments, and spatial adjacency constraints that limit the

maximum adjacent surface area to which clearcutting can be applied. The outcome was a highly complex problem that cannot be solved with an exact method and then required the design and implementation of a Multiobjective Metaheuristic, that we designed on the base of an Scatter Search method . The proposed model was applied to a real plantation belonging to the forestry company “Empresa Forestal Integral Macurije”, located in the region of Pinar del Río (Cuba). This research was published in [2].

On the other hand, the lack of integration among the factors involved in the wood sawing planning process creates a gap in the production system and, thus, technical-economic inefficiency in these industries. But, the predominant aim of a sawing plan has been to maximize production, although other important aspects are evident, such as performance, production level, warehousing availability and characteristics of raw material, etc. We developed a MO model to design a sawing plan guaranteeing demand for each product, generating as little waste as possible and efficiently managing the log stock, within the operative time constraints. The resulting Non-linear Integer MO problem was too hard to be solved with an exact solver, and so a multi-objective metaheuristic based on Scatter Search was adapted for its resolution. This model was used to solve a real case in Cuba. We were able to obtain a solution that satisfied demand with a waste rate below the desired level, minimized over-production, required less raw material from the forest and matched the machine time available in the sawmill. This was published in [1].

2.5 Optimal design of a Rough Sets grid. The basic ideas of rough sets based on dominance principles have been shown very useful to multicriteria classification problem in the literature. The problem with this kind of method is the high complexity of the underlying problem to find the optimal grid offering the best performance to the Rough Set method. In this project we have designed a Multiobjective Metaheuristic based on Tabu Search for the resolution of this problem, obtaining better results than a standard design over the most used set of test problem in the literature. This research was the main core of the Ph.D. dissertation of Iwona Gruszka (University of Katowice, Poland) and its publication is currently in preparation. On the other hand, the results obtained in the optimal design of this grid have been used also as a part of the Multiobjective Metaheuristic developed also in this project (Section 2.1).

3 Indicadores de resultados

Relating the attainment of our goals for the two first years (see Section 1), we completed all but the implementation of the Interactive Metaheuristic method for Multiobjective Programming, as described in Section 2. Although the root of this methodology is already designed and published (see Section 2.3), the final software implementation was displaced to the third year by the consecution of some goals for the third year (see Section 2.1 and Section 2.5), motivated by the finalization of two Ph.D. dissertations that made us slightly change our chronogram. This final implementation is now scheduled in the third year.

Relevance and originality of the results can be evaluated according to the quality of the journals where the results on the different goals were published (see Section 2), where most of them are JCR journals. On the other hand, all these goals have been productive, not only relating publications, but also Ph. D. dissertations and communications in relevant conferences, as MCDM,

GECCO, PPSN, CEC, etc. We have established collaborations with foreign and national researchers, like Carlos Coello (Mexico), Manuel Laguna (USA), Kaisa Miettinen (Finland) or Alfredo G. Hernández-Díaz (Seville). These collaborations still continue and conform the main root of our next project application, currently in preparation, including new applications coming from the economy, as Project Portfolio Selection, where we recently finished one more Ph.D. dissertation, to be added to the applied line we are working on, Forestry Management.

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